Exponential-Trigonometry Intuitionistic Fuzzy Divergence Measure

Rozy Boora, V.P. Tomar

***Abstract:** Based on the exponential and trigonometry functions, a new divergence measure is introduced under the intuitionistic fuzzy environment. The Intuitionistic Fuzzy Divergence Measure is an indispensable instrument to calculate the variance between two intuitionistic fuzzy sets. Many attractive properties are displayed to enhance the value of introduced intuitionistic fuzzy divergence measure. Many existing divergence measures are reviewed with their counter-intuitive examples. Finally, the invented divergence measure is applicative in the field of medical deliverance to differentiate between diseases with nearby same symptoms, pattern recognition to categorize the unknown into one of the known pattern.*

***Keywords:** Intuitionistic Fuzzy Sets, Fuzzy Sets, Divergence measure, Pattern Recognition, Medical Deliverance.*

I. INTRODUCTION

Information Theory was developed by Shannon [1] for handling the imprecise, uncertain data. To measure the uncertainty of the data, Shannon proposed the new term 'Entropy'. In an experiment Shannon defined entropy as
H(P)=$-\sum_{i=1}^{n} p_i ln\, p_i$
Where P=$\{p_1, p_2, \ldots, p_n\}$ is the probability distribution for the discrete random variable X. The directed divergence for two probability distributions P=$\{p_1, p_2, \ldots, p_n\}$ and Q=$\{q_1, q_2, \ldots, q_n\}$ was constructed as
D(P:Q)= $\sum_{i=1}^{n} p_i ln \frac{p_i}{q_i}$
by Kullback and Leibler [2].
Later on Zadeh [3] brought forth the idea of fuzzy sets which is useful in modeling the unpredictability of the data. Membership function depicts the Fuzzy Sets. Fuzzy Sets are incapable in modeling many practical situations. Since, then many generalities of the fuzzy sets have been developed such as Intuitionistic Fuzzy sets, Pythagorean fuzzy sets, spherical fuzzy sets, hesitant fuzzy sets, Interval-valued fuzzy sets, etc.

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As the generalities of fuzzy sets, Atanassov [4] proposed the concept of Intuitionistic fuzzy sets. Intuitionistic fuzzy sets are depicted by the non-membership function and membership function by which problems faced by fuzzy sets are overcomed. Intuitionistic fuzzy sets have far-reaching applications as it have applications in diverse domains such as medical deliverance, face recognition, speech recognition, pattern recognition, fuzzy air craft control, image processing, feature selection, bio-informatics, edge detection etc. The divergence measure is one of the notable tools of intuitionistic fuzzy sets by which, the disparity of two intuitionistic fuzzy sets can be calculated. Due to its wide applicability, many researchers have invented various divergence measures accompanying the applications. Vlachos and Sergiadis [5]was the first one to propose the divergence measure of two intuitionistic fuzzy sets and have shown the applicability in the domain of pattern recognition. Later on, various researchers have developed many divergence measures. Kaushik, Kumar and Bajaj [6] proposed a novel intuitionistic fuzzy divergence measure and an algorithm is provided to dredged up the edges of the image by the utilization of proposed divergence measure. Montes, Janis and Montes elucidated the axiomatic definition of the divergence measure in the intuitionistic fuzzy sets and constructed the methods to build the intuitionistic fuzzy divergence measure. Hung and Yang [7] accustomed the J-divergence measure in the theory of intuitionistic fuzzy sets, constructed some distance and intuitionistic fuzzy similarity measures using it and illustrated some numerical examples to show the performance of proposed J-divergence measure in the field of clustering and recognition of pattern. Verma and Sharma [8] proposed the divergence measure (relative information) as the generalization of divergence measure between intuitionistic fuzzy sets given by Wei and Ye [9] and shown its applicability in multi-criteria decision making. Montes, Pal, Janis and Montes [10] studied the link among intuitionistic fuzzy divergence, intuitionistic dissimilarity and intuitionistic fuzzy distances and proposed a framework to compare the intuitionistic fuzzy sets. Maheshwari and Srivastava [11] developed the divergence measure between two intuitionistic fuzzy sets on the basis of logarithm function and have applied the results in the field of medical deliverance. Verma and Maheshwari [12] constructed a novel divergence measure known as ' fuzzy Jensen-exponential divergence' and a procedure to resolve the problems of multi-criteria decision making and the utilized it on numerical examples. Zhang and Jiang [13] proposed the divergence measure in the intuitionistic fuzzy environment and demonstrated the results in the domain of pattern recognition and medical deliverance with the help of two numeric examples.

Garg, Aggarwal and Tripathi [14] developed a generalized parametric intuitionistic directed divergence measure by utilizing the notion of convex linear combinations of values of membership functions and illustrated examples of decision making. By utilizing the Jenson inequality and Shannon entropy, Joshi and Kumar [15] constructed a intuitionistic fuzzy divergence measure along with a new dissimilarity measure and a proposed a novel multiple attribute decision making method. Kumar and Joshi [16] constructed the divergence measure between the intuitionistic fuzzy sets known as intuitionistic fuzzy Jensen-Tsalli divergence measure in the presence of parameters and shown its applicability in the domain of medical deliverance and pattern recognition with the help of numerical examples. Ansari, Ansari and Mishra [17] developed a novel intuitionistic fuzzy divergence measure and a novel significant technique for edge detection. They also analogized the results obtained from proposed measure with the Chaira, Canny and Sobel methods. Kumar and Om parkash [18] prosposed the divergence measure between the intuitionistic fuzzy sets and established an algorithm which is useful in handling the faults in the turbine. Charward and Gitizadeh [19] introduced a novel exponential intuitionistic fuzzy divergence measure and a midterm framework for retaiers to know the best time-of-use electricity pricing by analzing load. Munde [20] developed the divergence measure between the intuitionistic fuzzy sets and validates its proof. Munde [21] developed the divergence measure between the intuitionistic fuzzy sets having two parameters and illustrated numerical to apply the results in decision-making. Joshi and Kumar [22] proposed the divergence measure between the intuitionistic fuzzy sets and have shown its applicability in medical deliverance and pattern recognition with the help of numerical examples. To determine the unknown weight information for alternatives in decision making methods, Liu, Li, Huang, Zhou and Zhang [23] contructed the similarity-divergence measure. They also designed an algorithm with the help of particle swarm optimization and demonstrated the methd to solve the numerical examples. Mishra et. al. [24] proposed the parametric divergence measure between the intuitionistic fuzzy sets and applied the results in health-care waste disposal alternative selection problem. Mishra, Kumari and Mishra [25] developed the Jensen-exponential divergence measure. They proposed a Multi-criteria decision making procedure to select the most suitable renewable energy resources among the other renewable energy resources. Ju [26] developed the divergence measure between the intuitionistic fuzzy sets and have shown its applicability in innovation management to resolve the decision-making problems. Thao [27] proposed the intuitionistic fuzzy divergence measure on the basis of Archimedean t-conorm operators and illustrated a numerical example to apply the results in multi-criteria decision-making. In the present communication, a novel intuitionistic fuzzy divergence measure on the basis of exponential and trigonometric functions. Arora and Tomar [28] proposed a novel parametric intuitionistic fuzzy divergence measure and studied some properties along with its applications in decision making in the field of medical deliverance.

II. PRELIMINARIES

Some fundamental definitions and operators associated with the fuzzy sets and intuitionistic fuzzy sets are discussed in this section.

Zadeh [29] developed the notion of fuzzy sets to tackle with the uncertainty of data and provided the definition of fuzzy sets which is as follows:

Definition 1. Let Ϛ=$\{\varsigma_1, \varsigma_2, \ldots \varsigma_n\}$ be the finite universal set. The fuzzy set A takes the form

A = $\{< \varsigma_i, \upsilon_A(\varsigma_i) >, \varsigma_i \in Ϛ\}$

Where υ_A: Ϛ→[0,1] denotes the membership function, $\upsilon_A(\varsigma)$ is the value of the associativity of the element of ς in A.

Atanassov [4] generalized the concept of fuzzy sets as intuitionistic fuzzy sets by the addition of non-membership function and described the definition of intuitionistic fuzzy set which is as follows:

Definition 2. Let Ϛ=$\{\varsigma_1, \varsigma_2, \ldots \varsigma_n\}$ be the finite universal set. An intuitionistic fuzzy set A takes the form A = $\{< \varsigma_i, \upsilon_A(\varsigma_i), \nu_A(\varsigma_i) >, \varsigma_i \in Ϛ\}$

Where υ_A, ν_A: Ϛ→[0,1] denotes the membership function and non-membership function respectively such that 0≤ $\upsilon_A(\varsigma_i) + \nu_A(\varsigma_i) \leq 1$, $\upsilon_A(\varsigma)$ and $\nu_A(\varsigma)$ is the value of membership and value of non-membership of the element of ς in A respectively. $\pi_A(\varsigma_i)$ denotes the degree of hesitation such that $\pi_A(\varsigma_i) = 1 - \upsilon_A(\varsigma_i) - \nu_A(\varsigma_i)$ for each $\varsigma_i \in Ϛ$.

Atanassov [30] described the following operators on intuitionistic fuzzy sets:

Let Ϛ=$\{\varsigma_1, \varsigma_2, \ldots \varsigma_n\}$be the finite universal set. Let A, B and C be three intuitionistic fuzzy sets defined on $\mathcal{X}$.

- **Complement of the intuitionistic fuzzy set A (A^c)**

A^c = $\{< \varsigma_i, \nu_A(\varsigma_i), \upsilon_A(\varsigma_i) >, \varsigma_i \in \mathcal{X}\}$

- **Intersection of the intuitionistic fuzzy sets (A ∩ B)**

A ∩ B = $\{< \varsigma_i, min\big(\upsilon_A(\varsigma_i), \upsilon_B(\varsigma_i)\big), max\big(\nu_A(\varsigma_i), \nu_B(\varsigma_i)\big) >, \varsigma_i \in Ϛ\}$

- **Union of the intuitionistic fuzzy sets (A ∪ B)**

A ∪ B = $\{< \varsigma_i, max\big(\upsilon_A(\varsigma_i), \upsilon_B(\varsigma_i)\big), min\big(\nu_A(\varsigma_i), \nu_B(\varsigma_i)\big) >, \varsigma_i \in Ϛ\}$

- **Inclusion Relation (A ⊆ B)**

A ⊆ B if and only if $\upsilon_A(\varsigma_i) \leq \upsilon_B(\varsigma_i)$ and $\nu_A(\varsigma_i) \geq \nu_B(\varsigma_i)$ $\forall\ \varsigma_i \in Ϛ$

Hung and Yang [31] provided the axiomatic definition of the intuitionistic fuzzy divergence measure $\Supset$ between A and B to discriminate the intuitionistic fuzzy sets A and B which is presented as follows:

($\Supset$1) 0≤ $\Supset(A|B) \leq 1$

($\Supset$2) $\Supset(A|B) = \Supset(B|A)$

($\Supset$3) $\Supset(A|B) = 0$ if and only if A = B

($\Supset$4) $\Supset(A|B) \leq \Supset(A|C)$ and $\Supset(B|C) \leq \Supset(A|C)$ if A ⊆ B ⊆ C, C is the intuitionistic fuzzy set.

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III. REVIEW OF EXISTING INTUITIONISTIC FUZZY DIVERGENCE MEASURES

Attracted by the importance of intuitionistic fuzzy divergence measure, many researchers have proposed various intuitionistic fuzzy divergence measures and demonstrated the applications in many domains such as pattern recognition, image processing, medical diagnosis, speech recognition etc. Some of the existing intuitionistic fuzzy divergence measures are reviewed in the present section. Some existing divergence measures between two intuitionistic fuzzy sets are as follows:

- **Vlachos and Sergiadis [5]**

  $\ni_{VS}(A|B) =$

  $$\sum_{i=1}^{n}\left[\begin{array}{l}\upsilon_A(\varsigma_i)ln\left(\frac{2\upsilon_A(\varsigma_i)}{\upsilon_A(\varsigma_i)+\upsilon_B(\varsigma_i)}\right)+\nu_A(\varsigma_i)ln\left(\frac{2\nu_A(\varsigma_i)}{\nu_A(\varsigma_i)+\nu_B(\varsigma_i)}\right)\\ \upsilon_B(\varsigma_i)ln\left(\frac{2\upsilon_B(\varsigma_i)}{\upsilon_A(\varsigma_i)+\upsilon_B(\varsigma_i)}\right)+\nu_B(\varkappa_i)ln\left(\frac{2\nu_B(\varsigma_i)}{\nu_A(\varsigma_i)+\nu_B(\varsigma_i)}\right)\end{array}\right]$$

- Similarly Hung and Yang [7], Verma and Sharma [8], Zhang and Jiang [13], Mao, Yao and Wang [32], Maheshwari and Srivastava [33], Maheshwari and Srivastava [11], Ohlan [34], Munde [21], Mishra , Kumari and Sharma [25], Nguyen Xuan Thao [27] and Arora and Tomar [28] provided the intuitionistic fuzzy divergence measures.

IV. COUNTER-INTUITIVE CASES

Some of the invented intuitionistic fuzzy divergence measures fails at particular examples. Some examples are given as follows

Example 3.1 Let A and B be two IFSs of $\mathcal{X}$ such that A= {< x_1, 0.44,0.385> , < x_2, 0.43,0.39 >,< x_3,0.42,0.38>}, B= {< x_1, 0.34,0.48> , < x_2, 0.37,0.46 >,< x_3,0.38,0.45>}

For the A and B Intuitionistic Fuzzy Divergence measures $\ni_{vs}$(A|B) becomes -0.0071 which is a negative value and violates the ∋1 axiom.

Example 3.2 Let A and B be two IFSs of $\mathcal{X}$ such that A= {< x_1, 0,0.5> , < x_2, 0.5,0 >,< x_3,0,0>}, B= {< x_1, 0.5,0.5>,< x_2, 0.5,0.5 >,< x_3,0.5,0>}

For the A and B Intuitionistic Fuzzy Divergence measures $\ni_{vs}$(A|B) becomes 0 which violates the ∋2 axiom.

Example 3.3 Let A and B be two IFSs of $\mathcal{X}$ such that A= {< x_1, 0,0.5> , < x_2, 0,0.5 >}, B= {< x_1, 0.5,0> , < x_2, 0.5,0 >}

For the A and B Intuitionistic Fuzzy Divergence measures $\ni_{ZJ}$(A|B) becomes 0 which violates the ∋2 axiom.

V. NEW PROPOSED INTUITIONISTIC FUZZY DIVERGENCE MEASURE

A novel intuitionistic fuzzy divergence measure is proposed on the basis of exponential and trigonometry functions.

Theorem 1 Consider the two intuitionistic fuzzy sets A and B defined on finite universal set Ϛ. Then the new invented intuitionistic fuzzy divergence measure is as follows

$$\ni(A|B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]}-\frac{1}{e} \quad \ldots(A)$$

Proof: Let Ϛ={$\varsigma_1, \varsigma_2, \ldots \varsigma_n$}be the finite universal set and A, B and C are three intuitionistic fuzzy sets such that A={< $\varsigma_i, \upsilon_A(\varsigma_i), \nu_A(\varsigma_i)$ >, $\varsigma_i \in$ Ϛ}, B={< $\varsigma_i, \upsilon_B(\varsigma_i), \nu_B(\varsigma_i)$ >, $\varsigma_i \in$ Ϛ}, C = {< $\varsigma_i, \upsilon_C(\varsigma_i), \nu_C(\varsigma_i)$ >, $\varsigma_i \in$ Ϛ } and $\upsilon_A(\varsigma_i), \nu_A(\varsigma_i), \pi_A(\varsigma_i), \upsilon_B(\varsigma_i), \nu_B(\varsigma_i), \pi_B(\varsigma_i), \upsilon_C(\varsigma_i), \nu_C(\varsigma_i), \pi_C(\varsigma_i) \in [0,1]$ where $\pi_A(\varsigma_i) = 1-\upsilon_A(\varsigma_i)-\nu_A(\varsigma_i), \pi_B(\varsigma_i) = 1-\upsilon_B(\varsigma_i)-\nu_B(\varsigma_i), \pi_C(\varsigma_i) = 1-\upsilon_C(\varsigma_i)-\nu_C(\varsigma_i)$

(∋1) Since $\upsilon_A(\varsigma_i), \nu_A(\varsigma_i), \pi_A(\varsigma_i), \upsilon_B(\varsigma_i), \nu_B(\varsigma_i), \pi_B(\varsigma_i), \upsilon_C(\varsigma_i), \nu_C(\varsigma_i), \pi_C(\varsigma_i) \in [0,1]$, then we have

$$0 \le \frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2} \le \frac{1}{2}$$

$$0 \le \frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2} \le \frac{1}{2}$$

$$0 \le \frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2} \le \frac{1}{2}$$

Adding the above three inequalities we get

$$0 \le \frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2} \le \frac{3}{2}$$

$$\Rightarrow \frac{\pi}{2}-\frac{1}{2} \le \frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right) \le \frac{\pi}{2}$$

$$\Rightarrow 0.36787 \le e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} \le 0.4158$$

$$\Rightarrow 0 \le e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]}-\frac{1}{e} \le 0.04793$$

$$\Longrightarrow 0 \le \ni(A|B) \le 1.$$

(∋2)

$$\ni(A|B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]}-\frac{1}{e}$$

$$= e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_B(\varsigma_i)}-\sqrt{\upsilon_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_B(\varsigma_i)}-\sqrt{\nu_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_B(\varsigma_i)}-\sqrt{\pi_A(\varsigma_i)}\right)^2}{2}\right)\right]}-\frac{1}{e}$$

$$= \ni(B|A).$$

(∋3) If A=B, then

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$$\Supset(A|A) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_A(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e} = 0$$

If $$\Supset(A|B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e} = 0$$

$$\Longrightarrow \sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right] = 1$$

$$\Longrightarrow \frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2} = \frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2} = \frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2} = 0$$

$\Longrightarrow \upsilon_A(\varsigma_i) = \upsilon_B(\varsigma_i),\ \nu_A(\varsigma_i) = \nu_B(\varsigma_i)$ for all $\varsigma_i \in Ϛ$.

$\Longrightarrow$ A=B

$\Supset(A|B) = 0 \Leftrightarrow$ A=B.

($\Supset$4) For A⊆B⊆C, we have

$$\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2} \le \frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_C(\varsigma_i)}\right)^2}{2}$$

$$\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2} \le \frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_C(\varsigma_i)}\right)^2}{2}$$

$$\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2} \le \frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_C(\varsigma_i)}\right)^2}{2}$$

Adding the above three inequalities we get

$$\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2} \le \frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_C(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_C(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_C(\varsigma_i)}\right)^2}{2}$$

$$\Rightarrow \sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_C(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_C(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_C(\varsigma_i)}\right)^2}{2}\right)\right] \le \sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]$$

$$\Rightarrow e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e} \le e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_C(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_C(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_C(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$\Longrightarrow \Supset(A|B) \le \Supset(A|C)$

Similarly we can prove that $\Supset(B|C) \le \Supset(A|C)$.

This validates that $\Supset(A|B)$ is a intuitionistic fuzzy divergence measure.

Proposed Divergence measure also holds many exciting properties which increases the efficiency of the measure. Many interesting properties of the proposed measure are listed in the next theorem.

Theorem 2

Let A, B, C be three intuitionistic fuzzy sets defined over the universal set $\mathcal{X}$. The proposed intuitionistic fuzzy divergence measure holds the properties which are mentioned as follows:

R1. $\Supset(A|A^C) = \Supset(A^C|A)$

R2. $\Supset(A|B) = \Supset(A^C|B^C)$

R3. $\Supset(A|B^C) = \Supset(A^C|B)$

R4. $\Supset(A|B) + \Supset(A^C|B) = \Supset(A^C|B^C) + \Supset(A|B^C)$

R5. $\Supset(A|A^C) = 0$ if and only if $\upsilon_A(\varkappa_i) = \nu_A(\varkappa_i), \varkappa_i \in \mathcal{X}$

R6. $\Supset(A \cup B|A \cap B) = \Supset(A|B)$

R7. $\Supset(A \cap B|B) \le \Supset(A|B)$

R8. $\Supset(A \cup B|B) \le \Supset(A|B)$

R9. $\Supset(A|A \cap B) = \Supset(B|A \cup B)$

R10. $\Supset(A|A \cup B) = \Supset(B|A \cap B)$

R11. $\Supset(A \cap C|B \cap C) \le \Supset(A|B)$, for any intuitionistic fuzzy set C.

R12. $\Supset(A \cup C|B \cup C) \le \Supset(A|B)$, for any intuitionistic fuzzy set C.

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Proof:

R1. Consider

$$\Supset(A|A^C) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\nu_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\upsilon_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_A(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= \Supset(A^C|A)$$

R2. Consider

$$\Supset(A^C|B^C) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= \Supset(A|B)$$

R3. Consider

$$\Supset(A^C|B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= \Supset(A|B^C)$$

R4. On utilizing the properties of **R2** and **R3** we get $(A|B)+\Supset(A^C|B) = \Supset(A^C|B^C)+\Supset(A|B^C)$

R5. Consider

$$\Supset(A|A^C) = 0$$

$$\Leftrightarrow e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\nu_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\upsilon_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_A(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e} = 0$$

$$\Leftrightarrow \left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\nu_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\upsilon_A(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_A(\varsigma_i)}\right)^2}{2}\right) = 0$$

$$\Leftrightarrow \frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\nu_A(\varsigma_i)}\right)^2}{2} = 0$$

$$\Leftrightarrow \upsilon_A(\varsigma_i) = \nu_A(\varsigma_i),\ \varsigma_i \in \varsigma$$

$\Longrightarrow\ \Supset(A|A^C) = 0$ if and only if $\upsilon_A(\varsigma_i) = \nu_A(\varsigma_i),\ \varsigma_i \in \varsigma$

R6. Consider

$$\Supset(A\cup B|A\cap B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_{A\cup B}(\varsigma_i)}-\sqrt{\upsilon_{A\cap B}(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_{A\cup B}(\varsigma_i)}-\sqrt{\nu_{A\cap B}(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{1-\upsilon_{A\cup B}(\varsigma_i)-\nu_{A\cup B}(\varsigma_i)}-\sqrt{1-\upsilon_{A\cap B}(\varsigma_i)-\nu_{A\cap B}(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\max(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i)}-\sqrt{\min(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\min(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}-\sqrt{\max(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}\right)^2}{2}+\frac{\left(\sqrt{1-\max(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i)-\min(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}-\sqrt{1-\min(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i)-\max(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= \Supset(A|B)$$

R7. Consider

$$\Supset(A\cap B|B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_{A\cap B}(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_{A\cap B}(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{1-\upsilon_{A\cap B}(\varsigma_i)-\nu_{A\cap B}(\varsigma_i)}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$= e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\min(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\max(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{1-\min(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i))-\max(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i))}-\sqrt{\pi_B(\varsigma_i)}\right)^2}{2}\right)\right]} - \frac{1}{e}$$

$$\leq \Supset(A|B)$$

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R8. On the steps of proof of **R7**, we can prove the property **R8.**

R9. Consider

$$\ni(A|A\cap B) = e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_{A\cap B}(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_{A\cap B}(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{1-\upsilon_{A\cap B}(\varsigma_i)-\nu_{A\cap B}(\varsigma_i)}\right)^2}{2}\right)\right]}-\frac{1}{e}$$

$$=e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\min(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i))}\right)^2}{2}+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\max(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}\right)^2}{2}+\frac{\left(\sqrt{\pi_A(\varsigma_i)}-\sqrt{1-\min(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i))-\max(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}\right)^2}{2}\right)\right]}-\frac{1}{e}$$

$$=e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_B(\varsigma_i)}-\sqrt{\max(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i))}\right)^2}{2}+\frac{\left(\sqrt{\nu_B(\varsigma_i)}-\sqrt{\min(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}\right)^2}{2}+\frac{\left(\sqrt{\pi_B(\varsigma_i)}-\sqrt{1-\max(\upsilon_A(\varsigma_i),\upsilon_B(\varsigma_i))-\min(\nu_A(\varsigma_i),\nu_B(\varsigma_i))}\right)^2}{2}\right)\right]}-\frac{1}{e}$$

$$=e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\left(\sqrt{\upsilon_B(\varsigma_i)}-\sqrt{\upsilon_{A\cup B}(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\nu_B(\varsigma_i)}-\sqrt{\nu_{A\cup B}(\varsigma_i)}\right)^2}{2}+\frac{\left(\sqrt{\pi_B(\varsigma_i)}-\sqrt{1-\upsilon_{A\cup B}(\varsigma_i)-\nu_{A\cup B}(\varsigma_i)}\right)^2}{2}\right)\right]}-\frac{1}{e}$$

$=\ni(B|A\cup B)$

R10. On the steps of proof of **R9** , we can prove the property **R10.**

R11. Consider

$$\left(\sqrt{\min(\upsilon_A(\varsigma_i),\upsilon_C(\varsigma_i))}-\sqrt{\min(\upsilon_B(\varsigma_i),\upsilon_C(\varsigma_i))}\right)^2\le\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2$$

$$\left(\sqrt{\max(\nu_A(\varsigma_i),\nu_C(\varsigma_i))}-\sqrt{\max(\nu_B(\varsigma_i),\nu_C(\varsigma_i))}\right)^2\le\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2$$

$$\left(\begin{array}{c}\sqrt{1-\min(\upsilon_A(\varsigma_i),\upsilon_C(\varsigma_i))-\max(\nu_A(\varsigma_i),\nu_C(\varsigma_i))}\\-\sqrt{1-\min(\upsilon_B(\varsigma_i),\upsilon_C(\varsigma_i))-\max(\nu_B(\varsigma_i),\nu_C(\varsigma_i))}\end{array}\right)^2\le\left(\begin{array}{c}\sqrt{1-\upsilon_A(\varsigma_i)-\nu_A(\varsigma_i)}-\\\sqrt{1-\upsilon_B(\varsigma_i)-\nu_B(\varsigma_i)}\end{array}\right)^2$$

Utilizing the above three inequalities we get

$$\left[\begin{array}{c}\frac{\left(\sqrt{\min(\upsilon_A(\varsigma_i),\upsilon_C(\varsigma_i))}-\sqrt{\min(\upsilon_B(\varsigma_i),\upsilon_C(\varsigma_i))}\right)^2}{2}\\+\frac{\left(\sqrt{\max(\nu_A(\varsigma_i),\nu_C(\varsigma_i))}-\sqrt{\max(\nu_B(\varsigma_i),\nu_C(\varsigma_i))}\right)^2}{2}\\+\frac{\left(\begin{array}{c}\sqrt{1-\min(\upsilon_A(\varsigma_i),\upsilon_C(\varsigma_i))-\max(\nu_A(\varsigma_i),\nu_C(\varsigma_i))}\\-\sqrt{1-\min(\upsilon_B(\varsigma_i),\upsilon_C(\varsigma_i))-\max(\nu_B(\varsigma_i),\nu_C(\varsigma_i))}\end{array}\right)^2}{2}\end{array}\right]\le\left[\begin{array}{c}\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\begin{array}{c}\sqrt{1-\upsilon_A(\varsigma_i)-\nu_A(\varsigma_i)}-\\\sqrt{1-\upsilon_B(\varsigma_i)-\nu_B(\varsigma_i)}\end{array}\right)^2}{2}\end{array}\right]$$

$$\Rightarrow e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\sqrt{\min(\upsilon_A(\varsigma_i),\upsilon_C(\varsigma_i))}-\sqrt{\min(\upsilon_B(\varsigma_i),\upsilon_C(\varsigma_i))}\right)^2}{2}\\+\frac{\left(\sqrt{\max(\nu_A(\varsigma_i),\nu_C(\varsigma_i))}-\sqrt{\max(\nu_B(\varsigma_i),\nu_C(\varsigma_i))}\right)^2}{2}\\+\frac{\left(\begin{array}{c}\sqrt{1-\min(\upsilon_A(\varsigma_i),\upsilon_C(\varsigma_i))-\max(\nu_A(\varsigma_i),\nu_C(\varsigma_i))}\\-\sqrt{1-\min(\upsilon_B(\varsigma_i),\upsilon_C(\varsigma_i))-\max(\nu_B(\varsigma_i),\nu_C(\varsigma_i))}\end{array}\right)^2}{2}\end{array}\right)\right]}-\frac{1}{e}\le e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\begin{array}{c}\sqrt{1-\upsilon_A(\varsigma_i)-\nu_A(\varsigma_i)}-\\\sqrt{1-\upsilon_B(\varsigma_i)-\nu_B(\varsigma_i)}\end{array}\right)^2}{2}\end{array}\right)\right]}-\frac{1}{e}$$

$$\Rightarrow e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\sqrt{\upsilon_{A\cap C}(\varsigma_i)}-\sqrt{\upsilon_{B\cap C}(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\sqrt{\nu_{A\cap C}(\varsigma_i)}-\sqrt{\nu_{B\cap C}(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\begin{array}{c}\sqrt{1-\upsilon_{A\cap C}(\varsigma_i)-\nu_{A\cap C}(\varsigma_i)}-\\\sqrt{1-\upsilon_{B\cap C}(\varsigma_i)-\nu_{B\cap C}(\varsigma_i)}\end{array}\right)^2}{2}\end{array}\right)\right]}-\frac{1}{e}\le e^{-\sin\left[\frac{\pi}{2}-\frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\sqrt{\upsilon_A(\varsigma_i)}-\sqrt{\upsilon_B(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\sqrt{\nu_A(\varsigma_i)}-\sqrt{\nu_B(\varsigma_i)}\right)^2}{2}\\+\frac{\left(\begin{array}{c}\sqrt{1-\upsilon_A(\varsigma_i)-\nu_A(\varsigma_i)}-\\\sqrt{1-\upsilon_B(\varsigma_i)-\nu_B(\varsigma_i)}\end{array}\right)^2}{2}\end{array}\right)\right]}-\frac{1}{e}$$

$\Rightarrow\ \ni(A\cap C|B\cap C)\le\ \ni(A|B)$

R12. On the similar steps of proof of property **R11,** property **R12** can be proved.

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VI. APPLICATIONS

Intuitionistic fuzzy Divergence measure has fascinated many researchers due to its wide range of applications in practical life. In this section, the authors have demonstrated the applicability of proposed intuitionistic fuzzy divergence measure in the field of medical diagnosis, pattern recognition.

A. Medical Diagnosis

In the practical situation, many people are suffering from diseases and it is the responsibility of the medical field to cure the disease by giving proper medication. Many diseases have common symptoms, so it becomes difficult for doctor to cure the patients with the proper medicine. In the field of medical deliverance, intuitionistic fuzzy divergence measure abets the doctors to give proper medication to the patients to cure the disease. The novel introduced intuitionistic fuzzy divergence measure aids the doctors to detect the disease from which the patient is suffering.

Let us consider a practical situation in which five patients Maddy, Bobby, Charliet, Vimal are put up with the diseases viral fever, malaria, typhoid, stomach problem, chest problem. The symptoms of the above mentioned diseases are temperature, headache, stomach pain, cough, Chest Pain. The particulars of the patients as described by Physicians in the form of non-membership values and membership values are recorded in the Table 1. Non-membership and the membership values of the symptoms of the diseases for the diagnosis are recorded in the Table 2.

We have considered the same problem as taken by [35] [36] which is described by the table 1 and table 2.

TABLE 1. Symptoms of the Patients

	Temperature	Headache	Stomach Pain	Cough	Chest Pain
Maddy	(0.8,0.1,0.1)	(0.6,0.1,0.3)	(0.2,0.8,0.0)	(0.6,0.1,0.3)	(0.1,0.6,0.3)
Bobby	(0.0,0.8,0.2)	(0.4,0.4,0.2)	(0.6,0.1,0.3)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Charliet	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.0,0.6,0.4)	(0.2,0.7,0.1)	(0.0,0.5,0.5)
Vimal	(0.6,0.1,0.3)	(0.5,0.4,0.1)	(0.3,0.4,0.3)	(0.7,0.2,0.1)	(0.3,0.4,0.3)

TABLE 2. Symptoms of the Diseases

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4,0.0,0.6)	(0.7,0.0,0.3)	(0.3,0.3,0.4)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Headache	(0.3,0.5,0.2)	(0.2,0.6,0.2)	(0.6,0.1,0.3)	(0.2,0.4,0.4)	(0.0,0.8,0.2)
Stomach Pain	(0.1,0.7,0.2)	(0.0,0.9,0.1)	(0.2,0.7,0.1)	(0.8,0.0,0.2)	(0.2,0.8,0.0)
Cough	(0.4,0.3,0.3)	(0.7,0.0,0.3)	(0.2,0.6,0.2)	(0.2,0.7,0.1)	(0.2,0.8,0.0)
Chest Pain	(0.1,0.7,0.2)	(0.1,0.8,0.1)	(0.1,0.9,0.0)	(0.2,0.7,0.1)	(0.8,0.1,0.1)

The values of the proposed Intuitionistic fuzzy divergence measure are calculated by utilizing the Table 1, Table 2 and proposed divergence measure and are recorded in the Table 3. Here '*' denotes the lowest value in the row. Lowest value of Intuitionistic fuzzy divergence measure donates the divergence of the given values from the exact values which are shown in terms of non-membership values and membership values. The disease from which the patients are suffering are recorded in the Table 4.

Table 3: Calculated Intuitionistic Fuzzy Divergence Measure

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Maddy	0.00036	0.00021	**0.00004***	0.00136	0.00182
Bobby	0.00087	0.00269	0.00029	**0.00003***	0.00103
Charliet	0.00043	0.00077	**0.00038***	0.00167	0.00276
Vimal	**0.00009***	0.00026	0.00027	0.00054	0.00110

Table 4: Diseases From Which Patients Are Suffering

Patient	**Maddy**	**Bobby**	**Charliet**	**Vimal**
Diseases	**Typhoid**	**Stomach Problem**	**Typhoid**	**Viral Fever**

B. Pattern Recognition

In this subsection, the authors have demonstrated the applications of the introduced intuitionistic fuzzy divergence measure in the field of pattern recognition.

Consider the n unknown patterns $P_1, P_2, \ldots, P_n$ having classifications $C_1, C_2, \ldots, C_n$ respectively. The unknown patterns are expressed as intuitionistic fuzzy sets defined on the universal set $\mathcal{X}=\{\varkappa_1, \varkappa_2, \ldots \varkappa_m\}$ as

$$P_i = \{\langle \varkappa_j, \upsilon_{P_i}(\varkappa_j), \nu_{P_i}(\varkappa_j)\rangle, \varkappa_j \in \mathcal{X}\}$$

Where $i = 1,2, \ldots, n$ and $j = 1,2, \ldots, m$.

Let us consider an unknown pattern Q which is expressed as intuitionistic fuzzy sets defined on the set $\mathcal{X}$ as

Q= $\{\langle \varkappa_j, \upsilon_Q(\varkappa_j), \nu_Q(\varkappa_j)\rangle, \varkappa_j \in \mathcal{X}\}$

Now, the target is to categorize the unknown pattern into one of the groups $C_1, C_2, \ldots, C_n$.

Compute the intuitionistic fuzzy divergence measure $\ni (P_n|Q), i = 1,2,..n$ between the unknown patterns and known pattern. Classify the unknown pattern Q to the group C_k where k is described by

$$k = arg\{min(\ni (P_1|Q),\ \ni (P_2|Q), \ldots, \ni (P_n|Q))\}.$$

Utilizing this algorithm the unknown pattern can be classified into one of the $C_1, C_2, \ldots, C_n$. Above algorithm is expressed in the form of following flowchart.

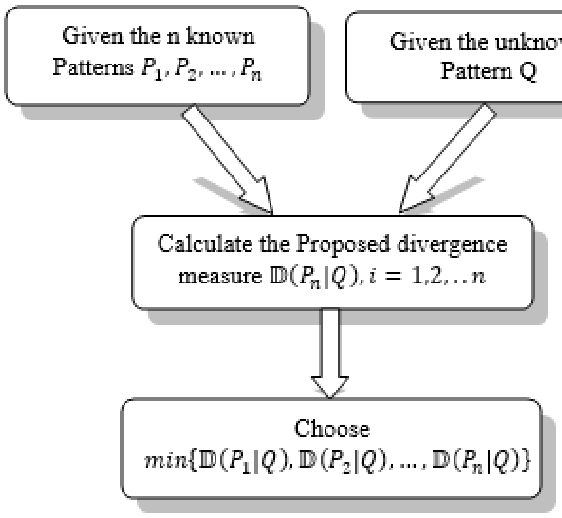

Figure 1. Flowchart representing the algorithm

A following practical example is provided in the field of recognition of pattern.

Example: Consider the three unknown patterns P_1, P_2 *and* P_3 having classifications C_1, C_2 and C_3 in the finite universal set $\mathcal{X}=\{\varkappa_1, \varkappa_2, \varkappa_3\}$. The unknown patterns are expressed as intuitionistic fuzzy sets as

$$P_1 = \{(\varkappa_1, 1.0,0.0), (\varkappa_2, 0.8,0.0), (\varkappa_3, 0.7,0.1)\},$$
$$P_2 = \{(\varkappa_1, 0.8,0.1), (\varkappa_2, 1.0,0.0), (\varkappa_3, 0.9,0.1)\},$$
$$P_3 = \{(\varkappa_1, 0.6,0.2), (\varkappa_2, 0.8,0.0), (\varkappa_3, 0.1,0.0)\}$$

Given the unknown pattern Q is expressed as intuitionistic fuzzy sets as

$$Q = \{(\varkappa_1, 0.5,0.3), (\varkappa_2, 0.6,0.2), (\varkappa_3, 0.8,0.1)\}$$

The target is to classify the unknown pattern into one of the groups C_1, C_2 and C_3.

Computed value of the novel introduced intuitionistic fuzzy divergence measures are $\ni (P_1|Q) = 0.000379$, $\ni (P_2|Q) = 0.000244$ and $\ni (P_3|Q) = 0.000108$. Utilizing the above algorithm, the authors can conclude that the unknown pattern Q should be classified in the group C_3.

VII. CONCLUSION

Intuitionistic fuzzy divergence measure is one of the noteworthy tools to compute the divergence of one intuitionistic fuzzy set from the other intuitionistic fuzzy sets. In the present communication, intuitionistic fuzzy divergence measure is introduced on the basis of exponential and trigonometry functions along with the corroboration for it. Some exciting properties are also described and proved to show the importance of intuitionistic fuzzy divergence measure. Finally, an example is demonstrated to apply the results in the domain of medical deliverance, pattern recognition. Further, the applications of the introduced intuitionistic fuzzy divergence measure can be illustrated in the domain of image segmentation, face recognition, edge detection, multi-criteria decision making etc.

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