Continuous Monotonic Decomposition of Some Standard Graphs by using an Algorithm

C. Sujatha, A. Manickam

***Abstract:** In this paper we elaborate an algorithm to compute the necessary and sufficient conditions for the continuous monotonic star decomposition of the bipartite graph $K_{m,r}$ and the number of vertices and the number of disjoint sets. Also an algorithm to find the tensor product of $P_n \times P_s$ has continuous monotonic path decomposition. Finally we conclude that in this paper the results described above are complete bipartite graphs that accept Continuous monotonic star decomposition. There are many other classes of complete tripartite graphs that accept Continuous monotonic star decomposition. In this research article Extended to complete m-partite graphs for grater values of m. Also the algorithm can be developed for the tensor product of different classes such as $C_n, W_n, K_{1,n}$ with P_n*

Keywords:** Complete bipartite graph, Continuous monotonic star decomposition, Tensor product. **Mathematical subject classification: *03B52, 03E72, 08A72.*

I. INTRODUCTION

A simple graph with the property that there is a path between every pair of vertices is known as a connected graph. The degree of a vertex *v* of any graph is the number of edges incident with *v* and is denoted by $d(v)$ and the distance between the two vertices *u* and *v* of *G* is the length of the shortest $v-u$ path in *G* and is denoted by $d(u,v)$. $\{G_i / i=1,2,\ldots n\}$ be a collection of edge-disjoint sub graphs of *G* such that $E(G)=E(G_1)\cup E(G_2)\cup\ldots\cup E(G_n)$, then the collection $\{G_i\}$ is called a decomposition of *G*. If each G_i is connected and $|E(G_i)|=i$ for each $i=1,2,\ldots,n$, then it is called a continuous monotonic decomposition of *G*. A complete graph with vertices $n\varepsilon N$, denoted by K_n is a connected simple graph with every vertex is connected with every other vertex by an edge [1]. A graph with *n* vertices $v_1,v_2,\ldots v_n$, where $n>=3$, and edges $\{v_1,v_2\}$ $\{v_1,v_2\},\{v_2,v_3\},\ldots,\{v_{n-1},v_n\},\{v_n,v_1\}$ is known as a cycle, C_n.

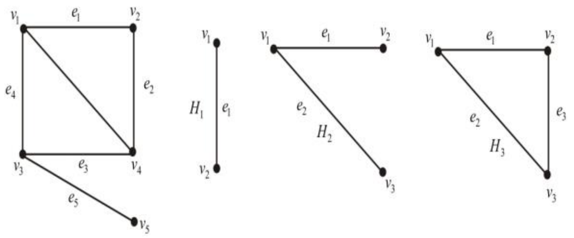

Fig.1.

II. STAR DECOMPOSITION OF A GRAPH

Let $G=(V,E)$ be a connected simple graph of order *m* and size *n*. If A decomposition $G_1,G_2,\ldots,G_n)$ of *G* is said to be a continuous monotonic decomposition (CMD) if each G_i is connected and $|E(G_i)|=i\ \forall i\varepsilon N$. [2].Introduced Ascending Sub graph Decomposition(ASD) as a decomposition of *G* into subgraphs G_i (not necessarily connected) and is isomorphic to a proper sub graph G_{i+1}. [3] Introduced a new concept known as continuous monotonic decomposition of Graphs [3]. If *G* admits a CMD, $\{G_1,G_2,\ldots,G_n\}|\ \forall n\varepsilon N$, where each G_i is a star, then we say that *G* admits Continuous Monotonic Cycle Decomposition (CMCD)[4].

III. CONTINUOUS MONOTONIC DECOMPOSITION OF COMPLETE BIPARTITE GRAPHS $K_{m,r}$

A graph *G* is a bipartite graph if $V(G)$ can be partitioned into two subsets *U* and *V*, called bipartite sets such that every edge of *G* joins a vertex of *u* and a vertex of *v*. In a bipartite graph, if every vertex of *u* is adjacent to every vertex of *v*, then such graph is called complete bipartite graph. A complete bipartite graph with $|U|=s$ and $|V|=t$ is denoted by $K_{s,t}$. If either $s=1$ or $t=1$ then $K_{s,t}$ is a star. Continuous Monotonic Decomposition of a wide variety of graphs had been studied by [4,5].

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IV. TENSOR PRODUCT

For two graphs *G* and *H*, the tensor product $G \times H$ has vertex set $V(G) \times V(H)$ in which (l_1, m_1) and (l_2, m_2) are adjacent whenever $l_1 l_2 \varepsilon E(G)$ and $m_1 m_2 \varepsilon E(H)$.

4.1. Theorem

Let *G* be a connected simple graph of order *r* and size *s*. Then *G* admits a CMD $H_1, H_2, \ldots, H_n$ if and only if $q = {}^{(n+1)}C_2$.

4.2. Theorem

K_n^+ admits a Continuous monotonic star decompositionfor all $n \geq 1$.

4.3. Theorem

(i) $K_{n,2n+1}$ admits Continuous monotonic star decomposition for $n \geq 1$

(ii) $K_{n+1,2n+1}$ admits Continuous monotonic star decomposition for $n \geq 1$

4.4. Theorem

There is an edge decomposition of *G* such that each partition class is a star in *G* if and only if *G* is bipartite.

4.5. Lemma

Let *G* be a edge disjoint union of stars $S_{i+1}, S_{i+2}, \ldots, S_{i+k}$ for some $k > 0$ such that $q = i(i+1)/2$. Then can be decomposed into stars $S_1, S_2, \ldots, S_i$.

4.6. Theorem

The complete bipartite graph $K_{m,r}(m \leq r)$ can be decomposed into stars $S_1, S_2, \ldots, S_{2n}$ (CMSD) if and only if $m = n-1$ and $r = 2n+1+j$ where $i, j > 0$ such that $n = i(j+1)/(j-2i)$.

4.6.1. Algorithm

1. $(K_{m,r})$

Step 1: Initially enter the values m, r, x

Step 2: Read the value of *x*

Step 3: for $i = 1$ to *n* do

Step 4: for $j = 1$ to *n* do

(i) if $(j > 2*i)$ then

(ii) compute $y = i(j+1)/(j-2*i)$

(iii) compute $m = y - i$

(iv) compute $r = 2*y+1+j$

Step 5: if $(m <= r \,\&\&\, m > 0)$

Step 6: Print i, j, y, m and *r*

Step 7: Go to step -3 until $i > n$

Step 8: Stop;

Step 9: End

Output

Enter the value of (*n*=15)

Table 1

I	*J*	*2n*	*M*	*R*
1	3	8	3	12
1	5	4	1	10
2	5	24	10	30
2	6	14	5	21
2	9	8	2	18
2	14	6	1	21
3	7	48	21	56
3	9	20	7	30
3	13	12	3	26
4	9	80	36	90
4	10	44	18	55
4	11	32	12	44
4	12	26	9	39
4	14	20	6	35
5	11	120	55	132
6	13	168	78	182
6	14	90	39	105

4.7. Continuous monotonic decomposition of Tensor Product of $P_n \times P_s$

4.7.1. Theorem

For any integer n, $P_n \times K_2$ has a CMD $\{H_1, H_2, \ldots, H_m\}$ if and only if there exist an integer *m* satisfying the following properties.

(i). $m = 4k$ or $m = 4k-1\,(k \geq 1, k\varepsilon Z)$

(ii) $m(m+1)/2 = 4n-4$

4.7.2. Lemma

(i) Let $m \equiv 0$ (mod 4). Two copies of the set $\{1,2,\ldots,m\}$ can be partitioned into four sets S_1, S_2, S_3 and S_4 such that $\Sigma_{a\varepsilon S_1} a = \Sigma_{b\varepsilon S_1} b = \Sigma_{c\varepsilon S_1} c = \Sigma_{d\varepsilon S_1} d = n-1.$ Here $m(m+1) = 4n-4.$

(ii) Let $m+1 \equiv 0$ (mod 4). Two copies of the set $\{1,2,\ldots,m\}$ can be partitioned into four sets S_1, S_2, S_3 and S_4 such that $\Sigma_{a\varepsilon S_1} a = \Sigma_{b\varepsilon S_1} b = \Sigma_{c\varepsilon S_1} c = \Sigma_{d\varepsilon S_1} d = n-1.$ Here $m(m+1) = 4n-4.$

4.7.3. Theorem

For any integer n, $P_n \times K_2$ has two copies of continuous monotonic decomposition $\{H_1, H_2, \ldots, H_m\}$ if and only if there exist an integer *m* satisfying the following properties.

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(i) $m = 4k$ or $m = 4k - 1\,(k \geq 1, k\varepsilon Z)$

(ii) $m(m+1) = 4n - 4$

Proof

Let $G = P_n \times P_3$. By definition $|E(G)| = 4n = 4$. Assume $P_n \times P_3$ has two copies of CMD $\{H_1, H_2, \ldots\ldots, H_m\}$. We have $|E(G)| = 2x^{(n+1)}C_2$. Here $4n - 4 = 2x^{(n+1)}C_2$. Since $P_n \times P_3$ has two copies of CMD, $4n - 4 = 2\times(1+2+\ldots\ldots+m) = 2\times m(m+1)/2 = m(m+1)$.

Hence $m(m+1) = 4k(K \geq 1,\, k\varepsilon Z)$. Now either $m = 4k$ or $m = 4k - 1$.

Conversely assume $m(m+1) = 0 \pmod 4$. Let $G = P_n \times P_3$. Let $P_n = (u_1, u_{2,} \ldots\ldots\ldots u_n)$, $P_3 = (v_1, v_2, v_3)$.

Define $w_{ij} = (u_i, v_j)$, where $1 \leq i \leq n,\ 1 \leq j \leq 3$.

Case 1: Suppose *n* is odd.

Define

$T_1 = \{w_{i1}, w_{(i+1)2} : 1 \leq i \leq n-1,\ i - odd\}$
$\cup \{w_{i2}, w_{(i+1)1} : 1 \leq i \leq n,\ i - even\}$.

$T_2 = \{w_{i2}, w_{(i+1)1} : 1 \leq i \leq n-1,\ i - odd\}$
$\cup \{w_{i1}, w_{(i+1)2} : 1 \leq i \leq n,\ i - even\}$.

$T_3 = \{w_{i2}, w_{(i+1)3} : 1 \leq i \leq n-1,\ i - odd\}$
$\cup \{w_{i3}, w_{(i+1)2} : 1 \leq i \leq n,\ i - even\}$.

$T_4 = \{w_{i3}, w_{(i+1)2} : 1 \leq i \leq n-1,\ i - odd\}$
$\cup \{w_{i2}, w_{(i+1)3} : 1 \leq i \leq n,\ i - even\}$.

Here $|T_1| = |T_2| = |T_3| = |T_4| = n - 1$. Also $|T_1| + |T_2| = {}^{(m+1)}C_2$ and $|T_3| + |T_4| = {}^{(m+1)}C_2$. By Lemma 10, $\{1,2,\ldots.,m\} = S_1 \cup S_2$ and $\{1,2,\ldots.,m\} = S_3 \cup S_4$

$\Sigma_{a\varepsilon S_1} a = \Sigma_{b\varepsilon S_1} b = \Sigma_{c\varepsilon S_1} c = \Sigma_{d\varepsilon S_1} d$. Decompose T_1, T_2 into trees $\{H_i\}$ as follows: $T_1 = \cup_{i\varepsilon S_1} H_i$ and $T_2 = \cup_{i\varepsilon S_2} H_i$. $|E(H_i)| = i$, where $1 \leq i \leq m$. Also decompose T_3, T_4 into trees $\{H_i\}$ as follows: $T_3 = \cup_{i\varepsilon S_3} H_i$ and $T_4 = \cup_{i\varepsilon S_4} H_i$. $|E(H_i)| = i$, where $1 \leq i \leq m$. Clearly two copies of $\{H_1, H_2, \ldots. H_m\}$ forms the continuous monotonic decompositions of $P_n \times P_3$.

Case 2: Suppose *n* is even.

Define T_1, T_2, T_3 and T_4. As the case 1, two copies of $\{H_1, H_2, \ldots\ldots, H_m\}$ forms the continuous monotonic decompositions of $P_n \times P_3$.

4.7.3.1. Algorithm

Step 1: Initially enter the values n, m, a, b

Step 2: Read the values of m, n

Step 3: While $(a \leq m - 1)$

Step 4: Store $a = 1;\ b = 2;$

(i) For $i = 1$ to n do

(ii) Line $(i, a) - (i+1, b)$

(iii) Swap (a, b)

Step 5: Store $a = 2; b = 1$

(i) For $i = 1$ to n do

(ii) Line $(i, a) - (i+1, b)$

(iii) Swap (a, b)

Step 6: Goto Step 3 Until $(a \leq m - 1)$

Step 7: Stop

Step 8: End

Output.

Table 2 : Tensor Product of $P_{15} \times P_3$

T_1		T_2		T_3		T_4	
From	**To**	**From**	**To**	**From**	**To**	**From**	**To**
(1, 1)	(2, 2)	(1, 2)	(2, 1)	(1, 2)	(2, 3)	(1, 3)	(2, 2)
(2, 2)	(3, 1)	(2, 1)	(3, 2)	(2, 3)	(3, 2)	(2, 2)	(3, 3)
(3, 1)	(4, 2)	(3, 2)	(4, 1)	(3, 2)	(4, 3)	(3, 3)	(4, 2)
(4, 2)	(5, 1)	(4, 1)	(5, 2)	(4, 3)	(5, 2)	(4, 2)	(5, 3)
(5, 1)	(6, 2)	(5, 2)	(6, 1)	(5, 2)	(6, 3)	(5, 3)	(6, 2)
(6, 2)	(7, 1)	(6, 1)	(7, 2)	(6, 3)	(7, 2)	(6, 2)	(7, 3)
(7, 1)	(8, 2)	(7, 2)	(8, 1)	(7, 2)	(8, 3)	(7, 3)	(8, 2)
(8, 2)	(9, 1)	(8, 1)	(9, 2)	(8, 3)	(9, 2)	(8, 2)	(9, 3)
(9, 1)	(10, 2)	(9, 2)	(10, 1)	(9, 2)	(10, 3)	(9, 3)	(10, 2)
(10, 2)	(11, 1)	(10, 1)	(11, 2)	(10, 3)	(11, 2)	(10, 2)	(11, 3)
(11, 1)	(12, 2)	(11, 2)	(12, 1)	(11, 2)	(12, 3)	(11, 3)	(12, 2)
(12, 2)	(13, 1)	(12, 1)	(13, 2)	(12, 3)	(13, 2)	(12, 2)	(13, 3)
(13, 1)	(14, 2)	(13, 2)	(14, 1)	(13, 2)	(14, 3)	(13, 3)	(14, 2)
(14, 2)	(15, 1)	(14, 1)	(15, 2)	(14, 3)	(15, 2)	(14, 2)	(15, 3)

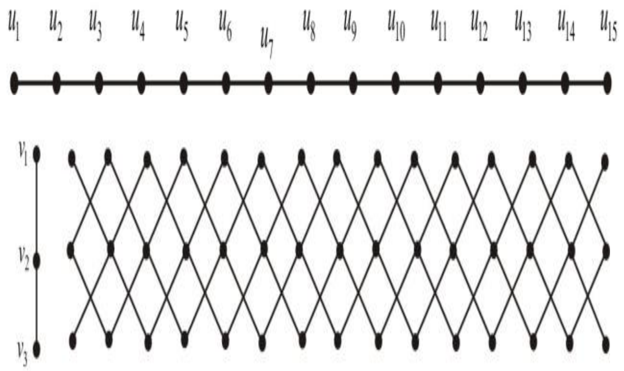

Fig.2.

V.CONCLUSION

In this paper we described above complete bipartite graphs that accept Continuous monotonic star decomposition**.** There are many other classes of complete tripartite graphs that accept Continuous monotonic star decomposition. Finally we conclude that in this paper can be extended to complete *m*-partite graphs for grater values of *m*. Also the algorithm can be developed for the tensor product of different classes such as $C_n, W_n, K_{1,n}$ with P_n.

CONFLICT OF INTEREST

The author confirms that there no conflict of interest to declare for this publications.

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Dr. C. Sujatha a well-known academician has taken the charge as principal of Marudupandiyar College, Thanjavur, Since December 2018. She had obtained her doctoral degree from Bharathidasan University Tiruchirappalli, which is ranked among the top world universities. She is a Mathematician by profession and specialized in Graph theory and applications. She has more than 15 years of teaching and 7 years of research experience. She has published many research articles in reputed in International Journals and conferences and also organized many Conference, Seminar and Workshops. She has the vision of fostering academic learning and research among students and research scholars for the development of innovative concepts and products to the betterment of society. She firmly believes that quality education can only lead to innovations and new product developments. She her wide and rich administrative research and international exposure to achieve the vision and Build Institutions.

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