Comparison of Haar Wavelet Collocation and Finite Element Methods for Solving the Typical Ordinary Differential Equations

S. C. Shiralashetti, P. B. Mutalik Desai, A. B. Deshi

Abstract - In this paper, we developed an efficient Haar Wavelet Collocation Method (HWCM) for solving typical Ordinary Differential Equations (ODE). In particular, it is shown that the computed results of HWCM are superior to Finite Element Method (FEM) as compared with the exact solution. The present study is illustrated by exploring different kinds of Typical Ordinary Differential Equations that shows the pertinent features of the Haar wavelet collocation method.

Keywords: Finite Element Method, Haar wavelet Collocation method, singular value and the study of Calderon-Zygmund operators in mathematics.

INTRODUCTION

The numerical solution of ordinary differential equations is one of the older, more established branches of numerical analysis. Yet, despite an abundance of methods to treat differential equations when the boundary conditions are known, at the present time, there does not exist a single numerical method for producing the general solution of a differential equation directly. As a result, there is a prevalent feeling among many scientists and engineers that while numerical methods provide useful information in specific cases, they are inferior to analytic methods which describe the behaviour of a system under arbitrary conditions. The Finite Element Method (FEM) means going from part to whole is an effective tool for numerical solutions to a large class of engineering problems. Many Researchers have contributed to the development of FEM [1-6] since its origin. Due to its diversity and flexibility, as an analysis tool FEM has attracted engineering and science education considerably. FEM will give approximate numerical solutions for complex industrial problems, where exact solutions are difficult to obtain. Some of the complex problems are cooling of electronic equipment, metal temperatures in the case of gas turbine blades, cooling problems in electrical mortars etc. The name Wavelet or Ondelette was introduced in the end of 1980 by French mathematicians. The existence of Wavelets and many ideas originated from work in sub band coding in engineering, coherent states and renormalization group theory in physics and the study of Calderon-Zygmund operators in mathematics.

The Haar Wavelets have gained popularity among researchers for their useful properties such as simple applicability, orthogonality and compact support. Due to the linear and piecewise nature, the Haar Wavelet basis laks differentiability and hence the integration approach will be used instead of the differentiation for calculation of the Coefficients [7-10]. The main concern of this paper is to introduce a Haar wavelet collocation and finite element methods for the solution of differential equations in which the dimension of the nullspace of a matrix representation of an ordinary differential operator is the same as the dimension of the nullspace of the operator itself. With these methods, the number of homogeneous solutions of the system of algebraic equations is equal to the number of homogeneous solutions of the original differential equation. Consequently, by evaluating the homogeneous solutions of the approximate system, and by also determining the particular integral, it is possible to obtain for the first time by the direct application of a numerical method, the approximate general solution of an ordinary differential equation. The objective of the study is to compare FEM andHWCM for solving the typical ODEs from the point of view of the formulation of the methods, describing the motivations that lead to them. Both of these methods have the ODE with variable coefficient as starting problem, where as the others are related with singular valued homogeneous, non homogeneous and nonlinear problems. The present work is organized as follows; Finite element method of solutions is presented in section 2. In section 3, Haar wavelets and Operational matrix of integration is discussed. Section 4 deals with the numerical findings with error analysis of test problems. Finally, conclusion of the proposed work is presented in section 5.

FINITE ELEMENT METHOD OF SOLUTIONS

Consider the differential equation to find the $u(t)$

$$-\frac{d}{dt}\left(a\frac{du}{dt}\right) + cu - f = 0, \text{For } 0 < t < 1 \quad (2.1)$$

Subjected to the boundary conditions

$$u(0) = u_0, \quad \left(a\frac{du}{dt}\right) = Q_0 \quad (2.2)$$

Where $a = a(t), \quad c = c(t), \quad f = f(x)$ here $u_0$ and $Q_0$ are given quantities of the problem.

We Seek an approximate solution to equation (2.1) over each finite element $\Omega$, is associated in the form
Comparison of Haar Wavelet Collocation and Finite Element Methods for Solving the Typical Ordinary Differential Equations

\[ u^e_h = \sum_{j=1}^n u_j^e \psi_j^e(t) \]  

(2.3)

Where \( u_j^e \) are the values of the solution, \( u(t) \) at the nodes of the finite element \( \Omega \), and \( \psi_j^e \) are the approximation functions over the element.

The \( i \)th algebraic equation of the system of \( n \) equations can be written as [4]

\[ 0 = \sum_{j=1}^n K_{ij}^e u_j^e - f_i^e - Q_i^e \quad (i=1, 2...n) \]  

(2.4)

Where

\[ K_{ij}^e = B^e(\psi_i^e, \psi_j^e) = \int_{x_i} \left( a \frac{d \psi_i^e}{dt} \frac{d \psi_j^e}{dt} + c \psi_i^e \psi_j^e \right) dt \]  

(2.5)

In Matrix notation the linear algebraic equations (2.4) can be written as

\[ [K^e] [u^e] = \{f^e\} + \{Q^e\} \]  

(2.6)

Here the matrix \( K^e \) is called the coefficient matrix or stiffness matrix. The column vector \( f^e \) is the source vector. \( u^e \) & \( Q^e \) called the primary and secondary variables.

The coefficient matrix and Column Vector are

\[ [K^e] = \frac{ae}{h_t} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{c_i h_e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \]

\[ \{f^e\} = \frac{a_i h_e}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]  

(2.7)

Imposing the Boundary conditions (2.2) on the given system of equations with \( f_i^e = 0 \), we get

\[ \begin{bmatrix} K_{11}^e & 0 & 0 & 0 \\ 0 & K_{12}^e & 0 & U_1 = 0 \\ 0 & K_{21}^e & K_{22}^e + K_{11}^e & K_{12}^e \\ 0 & 0 & K_{31}^e & K_{32}^e \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 = 0 \end{bmatrix} = \begin{bmatrix} Q_1^e \\ 2P^e \\ Q_2^e \\ Q_3^e \end{bmatrix} \]  

(2.8)

This Evolves four equations in four unknowns, \( U_3, Q_1^e, Q_2^e \) and \( Q_3^e \).

III. HAAR WAVELETS AND OPERATIONAL MATRIX OF INTEGRATION

The scaling function \( h_1(t) \) for the family of the Haar wavelets is defined as

\[ h_1(t) = \begin{cases} 1 & \text{for } t \in [0,1) \\ 0 & \text{otherwise} \end{cases} \]  

(3.1)

The Haar wavelet family for \( t \in [0,1) \) is defined as

\[ h_1(t) = \begin{cases} 1 & \text{for } t \in \left[ \frac{k}{m}, \frac{k+0.5}{m} \right) \\ -1 & \text{for } t \in \left[ \frac{k+0.5}{m}, \frac{k+1}{m} \right) \\ 0 & \text{otherwise} \end{cases} \]  

(3.2)

In the above definition the integer \( m = 2^l, \ l = 0, 1, ..., J \), indicates the level of resolution of the wavelet and integer \( k = 0, 1, ..., m-1 \) is the translation parameter.

Maximum level of resolution is \( J \). The index \( i \) in Eq. (3.2) is calculated using \( i = m + k + 1 \). In case of minimal values \( m = 1, k = 0 \) then \( i = 2 \). The maximal value of \( i \) is \( N = 2^{J+1} \).

Let us define the collocation points \( t_j = \frac{j - 0.5}{N} \), \( j = 1, 2, ..., N \), discretize the Haar function \( h_i(t) \), in this way, we get Haar coefficient matrix, \( H(i, j) = h_i(t_j) \) which has the dimension \( N \times N \). For instance, \( J = 3 \Rightarrow N = 16 \), then we have...
We establish an operational matrix for integration via Haar wavelets. The operational matrix of integration is obtained by integrating (2.2) is as,

\[ P_h = \int_0^t h'(t) \, dt \]  \hspace{1cm} (3.3)

and

\[ Q_h = \int_0^t P_h(t) \, dt \] \hspace{1cm} (3.4)

These integrals can be evaluated by using equation (2.2) and they are given by

\[
P_h_i(t) = \begin{cases} 
  \frac{t - k}{m} & \text{for } t \in \left[ \frac{k}{m}, \frac{k + 0.5}{m} \right] \\
  \frac{k + 1}{m} - t & \text{for } t \in \left[ \frac{k + 0.5}{m}, \frac{k + 1}{m} \right] \\
  0 & \text{otherwise}
\end{cases} \hspace{1cm} (3.5)
\]

\[
Q_h_i(t) = \begin{cases} 
  \frac{1}{2} \left( t - \frac{k}{m} \right)^2 & \text{for } t \in \left[ \frac{k}{m}, \frac{k + 0.5}{m} \right] \\
  \frac{1}{2} \left( \frac{k + 1}{m} - t \right)^2 & \text{for } t \in \left[ \frac{k + 0.5}{m}, \frac{k + 1}{m} \right] \\
  \frac{1}{4 m^2} & \text{for } t \in \left[ \frac{k + 1}{m}, 1 \right]
\end{cases} \hspace{1cm} (3.6)
\]

For instance, \( J = 3 \Rightarrow N = 16 \), from (3.5) then we have

\[
P_h(16, 16) = \frac{1}{\sqrt{2}} \begin{bmatrix}
  1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
  1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
  1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
  1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
and from (3.6) we get

\[
Q h(16,16) = \frac{1}{2048}
\]

IV. TEST PROBLEMS

Problem 1. Now, consider homogeneous differential equation with variable coefficient

\[
u'' + 4tu' + 2\left(1 + r^2\right)u = 0
\]

with the condition \( u(0) = 0, u(1) = 1 \) (4.2)

Case-1: FEM Solution: Comparing (4.1) with (2.1), we have

\[
p = -1, q = 4t, r = 2\left(1 + t^2\right)
\]

and \( s = 0 \), then from (2.7), then the coefficient matrix is

\[
K = K_{ij} = \int_{t_i}^{t_j} \frac{dL_i}{dt} \frac{dL_j}{dt} dt + 4\int_{t_i}^{t_j} L_i L_j dt + \int_{t_i}^{t_j} 2(1 + t^2)L_i L_j dt
\]

For two linear elements i.e., \( i = 1 \& 2 \), then

\[
L_i(t) = 1 - \frac{t}{h}, \quad L_2(t) = \frac{t}{h}, \quad \text{where} \ h = 1/M \text{ then we get}
\]

\[
K = \frac{-1}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + h \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} + \frac{h^3}{30} \begin{bmatrix} 2 & 3 \\ 3 & 12 \end{bmatrix}
\]

if \( M = 4 \), by the problem and conditions (4.2) and by assembling the matrix elements we get the matrix, after omitting first row, first column and last row, last column is

\[
\begin{bmatrix}
2 & 1 & 0 \\
-1 & 2 & 1 \\
0 & -1 & 2
\end{bmatrix} + \frac{h^3}{30} \begin{bmatrix}
14 & 3 & 0 \\
3 & 14 & 3 \\
0 & 3 & 14
\end{bmatrix} + \frac{1}{h} \begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix} \begin{bmatrix}
u_2 \\
u_3 \\
u_4
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
-4.2515
\end{bmatrix}
\]

then we get \( u_2 = 0.4231 \), \( u_3 = 0.7457 \) & \( u_4 = 0.9408 \).

Case-2: HWCM Solution:

Let us assume that

\[
u''(t) = \sum_{i=1}^{N} c_i h_i(t)
\]

By integrating (4.3) we have
\[ u'(t) = u'(0) + \sum_{i=1}^{N} c_i P_i(t) \]  \hspace{1cm} (4.4) \hspace{1cm} \text{Again}

Integrating (4.4) \( u(t) = u(0) + u'(0) t + \sum_{i=1}^{N} c_i Q_i(t) \)

Put \( t = 1 \), we get \( u'(0) = 1 - \sum_{i=1}^{N} c_i C_i(t) \) then

\[ u'(t) = 1 - \sum_{i=1}^{N} c_i C_i(t) + \sum_{i=1}^{N} c_i P_i(t) \]  \hspace{1cm} (4.5)

and

\[ u(t) = \left( 1 - \sum_{i=1}^{N} c_i C_i(t) \right) t + \sum_{i=1}^{N} c_i Q_i(t) \]  \hspace{1cm} (4.6)

where \( C_i = \int_{0}^{1} P_i(t) \, dt \) and for instance \( J = 3 \Rightarrow N = 16 \), then we have

\[
\begin{pmatrix}
128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 64 & 64 & 64 & 64 & 64 & 64 & 64 & 64 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 \\
0 & 0 & 0 & 0 & 72 & 72 & -28 & -28 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 32 & -4 & -4 & 4 & 4 & 4 & 4 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 8 & 4 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 8 & 4 & 4 \\
128 & -97 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 98 & -71 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 72 & -49 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 50 & -31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & -17 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1
\]

\[ C_{16,16} = \frac{1}{256} \]

Substituting (4.4), (4.4) and (4.6) in (4.1), we get

\[ \sum_{i=1}^{N} c_i h_i(t) + 4t \left( 1 - \sum_{i=1}^{N} c_i C_i(t) + \sum_{i=1}^{N} c_i P_i(t) \right) + 2(1+t^2) \left( 1 - \sum_{i=1}^{N} c_i C_i(t) \right) t + \sum_{i=1}^{N} c_i Q_i(t) = 0 \]  \hspace{1cm} (4.7)

Solving (4.7) using Inexact Newton’s method, we get the Haar wavelet coefficients \( c_i \) ’s =

\[-3.02, 0.35, 1.50, -0.97, 0.70, 0.57, -0.39, -0.50, 0.27, 0.42, 0.41, 0.14, -0.13, -0.25, -0.26, -0.23 \]

and the corresponding HWCM of the solution of (4.7) is obtained using the method presented in section 3 and is presented with FEM solution in the Table 1 for \( N=16 \) and Fig. 1 for \( N=32 \) in comparison with FEM and Exact solution \( u(t) = t \exp(-t^2 + 1) \). The error analysis for higher values of \( N \) is given in Table 2.

\[ u(t) = t \exp(-t^2 + 1) \]
Test Problem 2. Consider the homogeneous singular value problem,

\[ u'' + \frac{2}{t} u' - (4t^2 + 6)u = 0, \quad 0 < t \leq 1 \]

subject to conditions \( u(0) = 1, \quad u'(0) = 0 \)

Using the Procedure explained in section 2 & 3, we obtained the FEM and HWC methods solution and is compared with the exact solution \( y(x) = t^2 + t^3 \) is presented in fig 2. Its solution for N=32 is presented in Table 3 and its Error analysis is given in Table 4. FEM is not comparable with exact solution but HWC method gives comparable solution.

Test Problem 3. Now, consider the non homogeneous singular value problem

\[ u''(t) + \frac{2}{t} u'(t) + u(t) = 6 + 12t + t^2 + t^3 \]

subject to conditions \( u(0) = 1, \quad u'(0) = 0 \)

Using the Procedure explained in section 2 & 3, we obtained the FEM and HWC methods solution and is compared with the exact solution \( y(x) = \exp(t^2) \) is presented in fig 2 and the solution for N=32 is presented in Table 3 and its Error analysis is given in Table 4. FEM is not comparable with exact solution but HWC method gives comparable solution.

Test Problem 4. Lastly, consider the Non linear equation,

\[ u'' + \frac{2}{t} u' + u^5 = 0, \quad 0 < t \leq 1 \]

Case-1: FEM Solution: Comparing (4.25) with (3.1.1), we have,

\[ f_i = 0, \quad Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

and omitting the first row and first column, then we get, we obtained the solution as

\[ u_2 = -59.6677, \quad u_3 = -38.4862, \quad u_4 = -44.6500, \quad u_5 = -41.6969 \]

Case-2: HWCM Solution:

Let us assume that

\[ u''(t) = \sum_{i=1}^{N} a_i h_i(t) \]

By integrating (4.26) twice, we have

\[ u'(t) = \sum_{i=1}^{N} a_i P_i(t) \]
\[ u(t) = 1 + \sum_{i=1}^{N} a_i Q h_i(t) \]

\[(4.15)\]

Substituting (4.13)-(4.28) in (4.12), we get

\[ \sum_{i=1}^{N} a_i h_i(t) + \frac{2}{t} \sum_{i=1}^{N} a_i P h_i + \left(1 + \sum_{i=1}^{N} a_i Q h_i\right)^5 = 0 \]

\[(4.16)\]

Solving (4.29) using Inexact Newton’s method, we get the Haar coefficients \(C_i\)'s = [-0.22, -0.08, -0.03, -0.04, -0.01, -0.02, -0.02, -0.00, -0.01, -0.01, -0.01, -0.01, -0.01 & -0.01]. The obtained the numerical solution HWCM and FEM of (4.12) is presented in comparison with the exact solution \(u(t) = \left(1 + \frac{t^2}{3}\right)^{-1/2}\) in the Table 7 for \(N=16\) and Fig. 4 for \(N=32\). The error analysis for higher values of \(N\) is given in Table 8.

**V. CONCLUSION**

This paper presents a generalized procedure for FEM & HWCM for the solutions of some of ODEs were analyzed and their characteristics in terms of accuracy were examined. During the course of investigation, several new phenomena were explored. 1. Typical ODE with variable coefficient problems reveals that both FEM exhibits the non-comparable with exact solution. But HWCM gives the accurate solution as compared to exact. 2. As far as the singular valued ODEs are concerned, FEM is not comparable with exact solution but HWCM method gives comparable solution with true solution. 3. In case of non-linear ODE, HWCM gives excellent solutions than FEM as compared with exact solutions, which is justified.

**REFERENCES**


**Table1. Comparison of FEM and HWCM with Exact solutions for N=16 of the Test Problem 1.**

<table>
<thead>
<tr>
<th>t (=1/32)</th>
<th>FEM (F)</th>
<th>Exact (E)</th>
<th>HWCM (H)</th>
<th>Absolute Errors</th>
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<td></td>
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<td></td>
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Table 2. Error analysis of the Test Problem 1.

<table>
<thead>
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<th>N</th>
<th>$L_{\infty}$ (FEM)</th>
<th>MRE(FEM)</th>
<th>RPD(FEM)</th>
<th>$L_{\infty}$ (HWCM)</th>
<th>MRE(HWCM)</th>
<th>RPD(HWCM)</th>
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<td>0.1737</td>
<td>8.1832</td>
<td>0.3210</td>
</tr>
<tr>
<td>128</td>
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<td>37.6375</td>
<td>8.2639</td>
<td>0.1740</td>
<td>16.3873</td>
<td>0.3210</td>
</tr>
<tr>
<td>256</td>
<td>0.4002</td>
<td>75.3957</td>
<td>8.2820</td>
<td>0.1740</td>
<td>32.7844</td>
<td>0.3210</td>
</tr>
</tbody>
</table>

Fig. 1. Comparison of FEM & HWCM solution with exact solution for N=32 of Test Problem 1.

Table 3. Comparison of FEM and HWCM with Exact solutions for N=16 of the Test Problem 2.

| $t$ (=1/32) | FEM (F) 1.0e+03 * | Exact (E) | HWCM (H) | $|E - F|$ 1.0e+03 * | $|E - H|$ 1.0e-03 * |
|-------------|-------------------|-----------|----------|---------------|---------------|
| 1           | 5.6691            | 1.0010    | 1.0010   | 5.6681        | 0.0011        |
| 3           | 4.4235            | 1.0088    | 1.0088   | 4.4225        | 0.0014        |
| 5           | 4.2962            | 1.0247    | 1.0247   | 4.2952        | 0.0063        |
| 7           | 4.2933            | 1.0490    | 1.0490   | 4.2922        | 0.0137        |
| 9           | 4.3041            | 1.0823    | 1.0823   | 4.3031        | 0.0233        |
| 11          | 4.3166            | 1.1254    | 1.1254   | 4.3155        | 0.0350        |
| 13          | 4.3292            | 1.1794    | 1.1794   | 4.3280        | 0.0485        |
| 15          | 4.3419            | 1.2457    | 1.2457   | 4.3407        | 0.0634        |
| 17          | 4.3547            | 1.3261    | 1.3260   | 4.3533        | 0.0790        |
| 19          | 4.3674            | 1.4227    | 1.4226   | 4.3660        | 0.0942        |
| 21          | 4.3802            | 1.5383    | 1.5382   | 4.3787        | 0.1075        |
| 23          | 4.3931            | 1.6763    | 1.6762   | 4.3914        | 0.1167        |
| 25          | 4.4060            | 1.8411    | 1.8410   | 4.4041        | 0.1186        |
| 27          | 4.4189            | 2.0379    | 2.0378   | 4.4169        | 0.1083        |
| 29          | 4.4319            | 2.2734    | 2.2733   | 4.4296        | 0.0789        |
| 31          | 4.4449            | 2.5561    | 2.5561   | 4.4423        | 0.0206        |

Fig. 2. Comparison of FEM & HWCM solutions with exact solution for N=32 of Test Problem 2.
Table 2. Error analysis of the Test Problem 2.

<table>
<thead>
<tr>
<th>N</th>
<th>$L_{\infty}$ (FEM)</th>
<th>$L_{\infty}$ (HWCM)</th>
<th>$L_{rpd}$ (FEM)</th>
<th>$L_{rpd}$ (HWCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.3534e+03</td>
<td>1.3481e+03</td>
<td>3.0328e+07</td>
<td>3.7226e-04</td>
</tr>
<tr>
<td>16</td>
<td>5.6681e+03</td>
<td>5.6625e+03</td>
<td>5.2565e+08</td>
<td>1.1856e-04</td>
</tr>
<tr>
<td>32</td>
<td>2.3204e+04</td>
<td>2.3199e+04</td>
<td>8.7575e+09</td>
<td>3.1412e-05</td>
</tr>
<tr>
<td>64</td>
<td>9.3906e+04</td>
<td>9.3900e+04</td>
<td>1.4300e+11</td>
<td>7.9072e-06</td>
</tr>
<tr>
<td>128</td>
<td>3.7782e+05</td>
<td>3.7782e+05</td>
<td>2.3114e+12</td>
<td>1.9978e-06</td>
</tr>
<tr>
<td>256</td>
<td>1.5157e+06</td>
<td>1.5157e+06</td>
<td>3.7172e+13</td>
<td>4.9990e-07</td>
</tr>
</tbody>
</table>

Table 5. Comparison of FEM and HWCM with Exact solutions for N=16 of the Test Problem 3.

<table>
<thead>
<tr>
<th>N</th>
<th>$L_{\infty}$ (FEM)</th>
<th>$L_{\infty}$ (HWCM)</th>
<th>$L_{rpd}$ (FEM)</th>
<th>$L_{rpd}$ (HWCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>3.3011e+04</td>
<td>3.2779e+07</td>
<td>2.5893e+10</td>
<td>8.7271e-04</td>
</tr>
<tr>
<td>32</td>
<td>1.3157e+05</td>
<td>5.3063e+08</td>
<td>4.2927e+11</td>
<td>2.2143e-04</td>
</tr>
<tr>
<td>64</td>
<td>5.2530e+05</td>
<td>8.5939e+09</td>
<td>6.9955e+12</td>
<td>5.5747e-05</td>
</tr>
<tr>
<td>128</td>
<td>2.0992e+06</td>
<td>1.3704e+11</td>
<td>1.1298e+14</td>
<td>1.3983e-05</td>
</tr>
<tr>
<td>256</td>
<td>8.3927e+06</td>
<td>2.1958e+12</td>
<td>1.8161e+15</td>
<td>3.4967e-06</td>
</tr>
</tbody>
</table>

Table 6. Error analysis of the Test Problem 3.

<table>
<thead>
<tr>
<th>N</th>
<th>$L_{\infty}$ (FEM)</th>
<th>MRE(FEM)</th>
<th>RPD(FEM)</th>
<th>$L_{\infty}$ (HWCM)</th>
<th>MRE(HWCM)</th>
<th>RPD(HWCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>3.3011e+04</td>
<td>0.01007</td>
<td>0.01037</td>
<td>8.7271e-04</td>
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<td>2.8786e-05</td>
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<tr>
<td>32</td>
<td>1.3157e+05</td>
<td>0.02822</td>
<td>0.02895</td>
<td>5.3063e+08</td>
<td>0.8930</td>
<td>1.7937e-06</td>
</tr>
<tr>
<td>64</td>
<td>5.2530e+05</td>
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<td>0.05819</td>
<td>8.5939e+09</td>
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<tr>
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<td>1.3704e+11</td>
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<td>2.1958e+12</td>
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</tbody>
</table>

Fig. 3. Comparison of FEM & HWCM solution with exact solution for N=32 of Test Problem 3.
Comparison of Haar Wavelet Collocation and Finite Element Methods for Solving the Typical Ordinary Differential Equations

Table 7. Comparison of HWCM and FEM with Exact solutions for N=16 of the Test Problem 4.

<table>
<thead>
<tr>
<th>t (=1/32)</th>
<th>FEM (F) 1.0e+03 *</th>
<th>Exact (E)</th>
<th>HWCM (H)</th>
<th>Absolute Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.9125</td>
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<td>0.9998</td>
<td>3.9115</td>
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<tr>
<td>3</td>
<td>3.0417</td>
<td>0.9985</td>
<td>0.9985</td>
<td>3.0407</td>
</tr>
<tr>
<td>5</td>
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<td>0.9960</td>
<td>0.9960</td>
<td>2.9426</td>
</tr>
<tr>
<td>7</td>
<td>2.9315</td>
<td>0.9921</td>
<td>0.9921</td>
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</tr>
<tr>
<td>9</td>
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</tr>
<tr>
<td>11</td>
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<td>0.9809</td>
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<tr>
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<td>0.9653</td>
<td>2.9234</td>
</tr>
<tr>
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<td>27</td>
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<td>0.8990</td>
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<td>29</td>
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<td>0.8860</td>
<td>2.9135</td>
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<td>0.8728</td>
<td>0.8727</td>
<td>2.9121</td>
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</table>

Table 8. Error analysis of the Test Problem 4.

<table>
<thead>
<tr>
<th>N</th>
<th>FEM</th>
<th>HWCM</th>
</tr>
</thead>
<tbody>
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<td>L∞</td>
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<td>L∞</td>
<td>L∞</td>
</tr>
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<td>1.1364e+03</td>
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<tr>
<td>16</td>
<td>3.9115e+03</td>
<td>4.5166e+03</td>
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<tr>
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<td>1.5591e+04</td>
<td>1.8003e+04</td>
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<tr>
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<td>6.2248e+04</td>
<td>7.1877e+04</td>
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<td>2.8724e+05</td>
</tr>
<tr>
<td>256</td>
<td>9.9453e+05</td>
<td>1.1484e+06</td>
</tr>
</tbody>
</table>

Fig. 4. Comparison of FEM & HWCM solution with exact solution for N=32 of Test Problem 4.
Dr. S. C. Shiralashetti, was born in 1976. He received M.Sc., M.Phil, PGDCA, Ph.D. degree, in Mathematics from Karnataka University, Dharwad. He joined as a Lecturer in Mathematics in S. D. M. College of Engineering and Technology, Dharwad in 2000. Worked as a Assistant professor in Mathematics in Karnataka College Dharwad in 2009. From 2013 onwards working as a Associate Professor in the P.G. Department of studies in Mathematics, Karnataka University Dharwad. He has attended and presented More than 35 research articles in National and International Journals. He has published More than 36 research articles in National and International Journals and proceedings. Area of Research: Numerical Analysis, Wavelet Analysis, CFD, Differential Equations, Integral Equations, Integro-Differential Equations. H-index: 06; Citation index: 80; Ph.D. Working: 08; Research Projects Completed: 01; UGC-MRP (PI) (2013); Research Project Applied: UGC-MRP-01(2014); Life Member of the Association/Academy/Parishat/Society: 03; Awards: 04; Special Lectures Delivered: 20; Conference/Workshop organized as an organizing secretary / coordinator / Member: 10; Chaired the session in the National and International Conferences: 10; Conference/ Workshop/ Orientation/Refresher course attended: 23; Administrative Assignments completed: 02; Service given to the University in different capacities: 07.

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