Comparison of Haar Wavelet Collocation and Finite Element Methods for Solving the Typical Ordinary Differential Equations

S. C. Shiralashetti, P. B. Mutalik Desai, A. B. Deshi

Abstract - In this paper, we developed an efficient Haar Wavelet Collocation Method (HWCM) for solving typical Ordinary Differential Equations (ODE). In particular, it is shown that the computed results of HWCM are superior to Finite Element Method (FEM) as compared with the exact solution. The present study is illustrated by exploring different kinds of Typical Ordinary Differential Equations that shows the pertinent features of the Haar wavelet collocation method.

Keywords: Finite Element Method, Haar wavelet Collocation method, singular value Problems, Non-linear ODE.

I. INTRODUCTION

The numerical solution of ordinary differential equations is one of the older, more established branches of numerical analysis. Yet, despite an abundance of methods to treat differential equations when the boundary conditions are known, at the present time, there does not exist a single numerical method for producing the general solution of a differential equation directly. As a result, there is a prevalent feeling among many scientists and engineers that while numerical methods provide useful information in specific cases, they are inferior to analytic methods which describe the behaviour of a system under arbitrary conditions. The Finite Element Method (FEM) means going from part to whole is an effective tool for numerical solutions to a large class of engineering problems. Many Researchers have contributed to the development of FEM [1-6] since its origin. Due to its diversity and flexibility, as an analysis tool FEM has attracted engineering and science education considerably. FEM will give approximate numerical solutions for complex industrial problems, where exact solutions are difficult to obtain. Some of the complex problems are cooling of electronic equipment, metal temperatures in the case of gas turbine blades, cooling problems in electrical mortars etc. The name Wavelet or Ondelette was introduced in the end of 1980 by French mathematicians. The existence of Wavelets and many ideas originated from work in sub band coding in engineering, coherent states and renormalization group theory in physics and the study of Calderon-Zgymund operators in mathematics.

Manuscript Received on January 2015.
S. C. Shiralashetti, Department of Mathematics, Karnataka bniversity Dharwad, India.
P. B. Mutalik Desai, Department of Mathematics, K. L. E. College of Engineering and Technology, Chikodi, India.
A. B. Deshi, Department of Mathematics, Karnataka University Dharwad, India.

The Haar Wavelets have gained popularity among researchers for their useful properties such as simple applicability, orthogonality and compact support. Due to the linear and piecewise nature, the Haar Wavelet basis lakhs differentiability and hence the integration approach will be used instead of the differentiation for calculation of the Coefficients [7-10]. The main concern of this paper is to introduce a Haar wavelet collocation and finite element methods for the solution of differential equations in which the dimension of the nullspace of a matrix representation of an ordinary differential operator is the same as the dimension of the nullspace of the operator itself. With these methods, the number of homogeneous solutions of the system of algebraic equations is equal to the number of homogeneous solutions of the original differential equation. Consequently, by evaluating the homogeneous solutions of the approximate system, and by also determining the particular integral, it is possible to obtain for the first time by the direct application of a numerical method, the approximate general solution of an ordinary differential equation. The objective of the study is to compare FEM &HWCM for solving the typical ODEs from the point of view of the formulation of the methods, describing the motivations that lead to them. Both of these methods have the ODE with variable coefficient as starting problem, where as the others are related with singular valued homogeneous, non homogeneous and nonlinear problems. The present work is organized as follows; Finite element method of solutions is presented in section 2. In section 3, Haar wavelets and Operational matrix of integration is discussed. Section 4 deals with the numerical findings with error analysis of test problems. Finally, conclusion of the proposed work is presented in section 5.

II. FINITE ELEMENT METHOD OF SOLUTIONS

Consider the differential equation to find the $u(t)$

$$-\frac{d}{dt}\left(a\frac{du}{dt}\right)+cu-f=0, \text{ For } 0<t<1 \qquad (2.1)$$

Subjected to the boundary conditions

$$u(0)=u_0, \quad \left(a\frac{du}{dt}\right)=Q_0 \qquad (2.2)$$

Where $a=a(t)$, $c=c(t)$, $f=f(x)$ here u_0 and Q_0 are given quantities of the problem.

We Seek an approximate solution to equation (2.1) over each finite element Ω_e is associated in the form

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

$$u_h^e = \sum_{j=1}^{n} u_j^e \psi_j^e(t)$$
(2.3)

Where u_j^e are the values of the solution, $u(t)$ at the nodes of the finite element Ω_e and ψ_j^e are the approximation functions over the element.

The i[th] algebraic equation of the system of *n* equations can be written as [4]

$$0 = \sum_{j=}^{n} K_{ij}^e u_j^e - f_i^e - Q_i^e \qquad \text{(i=1, 2…n)}$$
(2.4)

Where

$$K_{ij}^e = B^e\left(\psi_i^e, \psi_j^e\right) = \int_{x_a}^{x_b}\left(a\frac{d\psi_i^e}{dt}\frac{d\psi_j^e}{dt} + c\psi_i^e\psi_j^e\right)dt$$

$$f_i^e = \int_{x_a}^{x_b} f\psi_i^e dt \qquad (2.5)$$

In Matrix notation the linear algebraic equations (2.4) can be written as

$$\left[K^e\right]\left\{u^e\right\} = \left\{f^e\right\} + \left\{Q^e\right\} \text{ Or } K^e u^e = f^e + Q^e$$
(2.6)

Here the matrix K^e is called the coefficient matrix or stiffness matrix. The column vector f^e is the source vector. u^e & Q^e called the primary and secondary variables.

The coefficient matrix and Column Vector are

$$\left[K^e\right] = \frac{ae}{h_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{c_e h_e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$

$$\left\{f^e\right\} = \frac{a_e h_e}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \qquad (2.7)$$

Imposing the Boundary conditions (2.2) on the given system of equations with $f_i^e = 0$, we get

$$\begin{bmatrix} K_{11}^1 & 0 & K_{12}^1 & 0 \\ 0 & K_{11}^2 & K_{12}^2 & 0 \\ K_{21}^1 & K_{21}^2 & K_{22}^1 + K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{bmatrix}\begin{Bmatrix} U_1 = 0 \\ U_2 = 0 \\ U_3 \\ U_4 = 0 \end{Bmatrix} = \begin{Bmatrix} Q_1^1 \\ Q_1^2 \\ 2P \\ Q_2^3 \end{Bmatrix}$$

(2.8)

This Evolves four equations in four unknowns, U_3 Q_1^1, Q_1^2 and Q_2^3.

III. HAAR WAVELETS AND OPERATIONAL MATRIX OF INTEGRATION

The scaling function $h_1(t)$ for the family of the Haar wavelets is defined as

$$h_1(t) = \begin{cases} 1 & for\ t \in [0,1) \\ 0 & otherwise \end{cases}$$

(3.1)

The Haar wavelet family for $t \in [0,1)$ is defined as

$$h_i(t) = \begin{cases} 1 & for\ t \in \left[\frac{k}{m}, \frac{k+0.5}{m}\right) \\ -1 & for\ \ t \in \left[\frac{k+0.5}{m}, \frac{k+1}{m}\right) \\ 0 & otherwise \end{cases}$$

(3.2)

In the above definition the integer $m = 2^l$, $l = 0,1,\ldots,J$, indicates the level of resolution of the wavelet and integer $k = 0,1,\ldots,m-1$ is the translation parameter.

Maximum level of resolution is J. The index i in Eq. (3.2) is calculated using $i = m + k + 1$. In case of minimal values $m = 1,\ k = 0$ then $i = 2$. The maximal value of i is $N = 2^{J+1}$.

Let us define the collocation points $t_j = \frac{j-0.5}{N}$, $j = 1,2,\ldots,N$, discretize the Haar function $h_i(t)$, in this way, we get Haar coefficient matrix, $H(i,j) = h_i(t_j)$ which has the dimension $N \times N$. For instance, $J = 3 \Rightarrow N = 16$, then we have

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

$$H(16,16)=\begin{pmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1\\
1&1&1&1&-1&-1&-1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&1&1&-1&-1&-1&-1\\
1&1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&-1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&-1&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&-1&-1\\
1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1
\end{pmatrix}$$

We establish an operational matrix for integration via Haar wavelets. The operational matrix of integration is obtained by integrating (2.2) is as,

$$Ph_i=\int_0^t h_i(t)dt \tag{3.3}$$

and

$$Qh_i=\int_0^t Ph_i(t)dt \tag{3.4}$$

These integrals can be evaluated by using equation (2.2) and they are given by

$$Ph_i(t)=\begin{cases} t-\dfrac{k}{m} & for\ t\in\left[\dfrac{k}{m},\dfrac{k+0.5}{m}\right) \\ \dfrac{k+1}{m}-t & for\ \ t\in\left[\dfrac{k+0.5}{m},\dfrac{k+1}{m}\right) \\ 0 & otherwise \end{cases} \tag{3.5}$$

$$Qh_i(t)=\begin{cases} \dfrac{1}{2}\left(t-\dfrac{k}{m}\right)^2 & for\ \ t\in\left[\dfrac{k}{m},\dfrac{k+0.5}{m}\right) \\ \dfrac{1}{4m^2}-\dfrac{1}{2}\left(\dfrac{k+1}{m}-t\right)^2 & for\ \ t\in\left[\dfrac{k+0.5}{m},\dfrac{k+1}{m}\right) \\ \dfrac{1}{4m^2} & for\ \ t\in\left[\dfrac{k+1}{m},1\right) \\ 0 & Otherwise \end{cases} \tag{3.6}$$

For instance, $J=3\Rightarrow N=16$, from (3.5) then we have

$$Ph(16,16)=\frac{1}{32}\begin{pmatrix}
1&3&5&7&9&11&13&15&17&19&21&23&25&27&29&31\\
1&3&5&7&9&11&13&15&15&13&11&9&7&5&3&1\\
1&3&5&7&7&5&3&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&3&5&7&7&5&3&1\\
1&3&3&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&3&3&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&3&3&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&3&3&1\\
1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1
\end{pmatrix}$$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

and from (3.6) we get

$$Qh(16,16)=\frac{1}{2048}\begin{pmatrix}
1 & 9 & 25 & 49 & 81 & 121 & 169 & 225 & 289 & 361 & 441 & 529 & 625 & 729 & 841 & 961\\
1 & 9 & 25 & 49 & 81 & 121 & 169 & 225 & 287 & 343 & 391 & 431 & 463 & 487 & 503 & 511\\
1 & 9 & 25 & 49 & 79 & 103 & 119 & 127 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 9 & 25 & 49 & 79 & 103 & 119 & 127\\
1 & 9 & 23 & 31 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32\\
0 & 0 & 0 & 0 & 1 & 9 & 23 & 31 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 9 & 23 & 31 & 32 & 32 & 32 & 32\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 9 & 23 & 31\\
1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7
\end{pmatrix}$$

IV. TEST PROBLEMS

Problem 1. Now**,** consider homogeneous differential equation with variable coefficient

$$u''+4tu'+2\left(1+t^2\right)u=0 \qquad (4.1)$$

with the condition $u(0)=0,\ u(1)=1$ (4.2)

Case-1: FEM Solution: Comparing (4.1) with (2.1), we have $p=-1, q=4t$, $r=2\left(1+t^2\right)$ and $s=0$, then from (2.7), then the coefficient matrix is

$$K=K_{ij}=\int_{t_a}^{t_b} t\frac{dL_i}{dt}\frac{dL_j}{dt}dt+4\int_{t_a}^{t_b} tL_i\frac{dL_j}{dt}dt+\int_{t_a}^{t_b} 2(1+t^2)L_iL_j dt$$

For two linear elements i.e. $i,\ j=1\&2$, then $L_1(t)=1-\frac{t}{h}\ \&\ L_2(t)=\frac{t}{h}$, where $h=1/M$ then we get

$K=\frac{-1}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+h\begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}+\frac{h^3}{30}\begin{bmatrix}2 & 3\\ 3 & 12\end{bmatrix}$, if M=4, by the problem and conditions (4.2) and by assembling the matrix elements we get the matrix, after omitting first row, first column and last row, last column is

$$\left(h\begin{bmatrix}2 & 1 & 0\\ -1 & 2 & 1\\ 0 & -1 & 2\end{bmatrix}+\frac{h^3}{30}\begin{bmatrix}14 & 3 & 0\\ 3 & 14 & 3\\ 0 & 3 & 14\end{bmatrix}+\frac{1}{h}\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}\right)\begin{Bmatrix}u_2\\ u_3\\ u_4\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ -4.2515\end{Bmatrix}$$

then we get $u_2=0.4231\ u_3=0.7457\ \&\ u_4=0.9408$.

Case-2: HWCM Solution:

Let us assume that
$$u''(t)=\sum_{i=1}^{N}c_ih_i(t) \qquad (4.3)$$

By integrating (4. 3) we have

$$u'(t) = u'(0) + \sum_{i=1}^{N} c_i Ph_i(t) \qquad (4.4)$$

Again integrating (4.4) $u(t) = u(0) + u'(0)t + \sum_{i=1}^{N} c_i Qh_i(t)$

Put $t = 1$, we get $u'(0) = 1 - \sum_{i=1}^{N} c_i Ch_i(t)$ then

$$u'(t) = 1 - \sum_{i=1}^{N} c_i Ch_i(t) + \sum_{i=1}^{N} c_i Ph_i(t) \qquad (4.5)$$

and
$$u(t) = \left(1 - \sum_{i=1}^{N} c_i Ch_i(t)\right) t + \sum_{i=1}^{N} c_i Qh_i(t) \qquad (4.6)$$

where $Ch_i = \int_0^1 Ph_i(t)\,dt$ and for instance $J = 3 \Rightarrow N = 16$, then we have

$$Ch(16,16) = \frac{1}{256}\begin{pmatrix}
128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\
128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 64 & 64 & 64 & 64 & 64 & 64 & 64 & 64 \\
128 & 128 & 128 & 128 & -16 & -16 & -16 & -16 & 16 & 16 & 16 & 16 & 16 & 16 & 16 & 16 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 32 & 32 & 32 & 16 & 16 & 16 & 16 \\
128 & 128 & -68 & -68 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
0 & 0 & 0 & 0 & 72 & 72 & -28 & -28 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 32 & -4 & -4 & 4 & 4 & 4 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 8 & 4 & 4 \\
128 & -97 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 98 & -71 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 72 & -49 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 50 & -31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & -17 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & -7 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1
\end{pmatrix}$$

Substituting (4.4), (4.4) and (4.6) in (4.1), we get

$$\sum_{i=1}^{N} c_i h_i(t) + 4t\left(1 - \sum_{i=1}^{N} c_i Ch_i(t) + \sum_{i=1}^{N} c_i Ph_i(t)\right) + 2(1+t^2)\left(\left(1 - \sum_{i=1}^{N} c_i Ch_i(t)\right) t + \sum_{i=1}^{N} c_i Qh_i(t)\right) = 0 \qquad (4.7)$$

Solving (4.7) using Inexact Newton's method, we get the Haar wavelet coefficients c_i 's =

[-3.02, 0.35, 1.50, -0.97, 0.70, 0.57, -0.39, -0.50, 0.27, 0.42, 0.41, 0.14, -0.13, -0.25, -0.26, -0.23] and the corresponding HWCM of the solution of (4.7) is obtained using the method presented in section 3 and is presented with FEM solution in the Table 1 for N=16 and Fig. 1 for N=32 in comparison with FEM and Exact solution $u(t) = t\ \exp(-t^2 + 1)$. The error analysis for higher values of N is given in Table 2.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

Test Problem 2. Consider the homogeneous singular value problem,

$$u''+\frac{2}{t}u'-(4t^2+6)u=0,\ 0<t\le 1$$

4.8)

Subjected to $u(0)=1,\ u'(0)=0$

(4.9)

Using the Procedure explained in section 2 & 3, we obtained the FEM and HWC methods solution and is compared with the exact solution $y(x)=\exp(t^2)$ is presented in fig 2 and the solution for N=32 is presented in Table 3 and Its Error analysis is given in Table 4. FEM is not comparable with exact solution but HWC method gives comparable solution.

Test Problem 3. Now, consider the non homogeneous singular value problem

$$u''(t)+\frac{2}{t}u'(t)+u(t)=6+12t+t^2+t^3$$

(4.10)

subject to conditions $u(0)=1,\ u'(1)=5$

(4.11)

Using the Procedure explained in section 2 & 3, we obtained the FEM and HWC methods solution and is compared with the exact solution $y(x)=t^2+t^3$ is presented in fig 3. Its solution for N=32 is presented in Table 5 and Its Error analysis is given in Table 6. FEM is not comparable with exact solution but HWC method gives comparable solution.

Test Problem 4. Lastly, consider the Non linear equation,

$$u''+\frac{2}{t}u'+u^5=0,\ 0<t\le 1$$

(4.12)

Case-1: FEM Solution: Comparing (4.25) with (3.1.1), we have, $p=t$, $q=-2$, $r=-tu^4$ and $s=0$, then from (3.1.5), then the coefficient matrix is

$$K=K_{ij}=\int_0^h\left[t\frac{dL_i}{dt}\frac{dL_j}{dt}-2L_i\frac{dL_j}{dt}-t(u_1L_1+u_2L_2)^4(L_iL_j)\right]dt$$

For two linear elements i.e. $i,j=1\&2$, then $L_1(t)=1-\frac{t}{h}\ \&\ L_2(t)=\frac{t}{h}$, where *h=1/M* then we get if *M*=4, assembling the *K* matrix elements we get,

$$\frac{1}{2}\begin{bmatrix}3&-3&0&0&0\\1&2&-3&0&0\\0&1&2&-3&0\\0&0&1&2&-3\\0&0&0&1&-1\end{bmatrix}-\frac{h^2}{840}\begin{bmatrix}70&70&0&0&0\\70&280&70&0&0\\0&70&280&70&0\\0&0&70&280&70\\0&0&0&70&210\end{bmatrix}$$

By the problem and conditions (4.14), $f_i=0$, $Q=[0,0,0,0,-0.5925]^T$ and omitting the first row and first column, then we get, we obtained the solution as $u_2=$ -59.6677, $u_3=$ -38.4862 $u_4=$ - 44.6500 $u_5=$ -41.6969

Case-2: HWCM Solution:

Let us assume that

$$u''(t)=\sum_{i=1}^{N}a_ih_i(t) \qquad (4.13)$$

By integrating (4.26) twice, we have

$$u'(t)=\sum_{i=1}^{N}a_iPh_i(t)$$

(4.14)

$$u(t)=1+\sum_{i=1}^{N} a_i Qh_i(t)$$

(4.15)

Substituting (4.13)-(4.28) in (4.12), we get

$$\sum_{i=1}^{N} a_i h_i(t)+\frac{2}{t}\sum_{i=1}^{N} a_i Ph_i+\left(\left(1+\sum_{i=1}^{N} a_i Qh_i\right)\right)^5=0$$

(4.16)

Solving (4.29) using Inexact Newton's method, we get the Haar coefficients c_i 's = [-0.22,

-0.08, -0.03, -0.04, -0.01, -0.02, -0.02, -0.02, -0.00, -0.01, -0.01, -0.01, -0.01, -0.01, -0.01 & -0.01]. The obtained the numerical solution HWCM and FEM of (4.12) is presented in comparison with the exact solution $u(t)=(1+\frac{t^2}{3})^{-1/2}$ in the Table 7 for N=16 and Fig. 4 for N=32. The error analysis for higher values of N is given in Table 8.

V. CONCLUSION

This paper presents a generalized procedure for FEM & HWCM for the solutions of some of ODEs were analyzed and their characterises in terms of accuracy were examined. During the course of investigation, several new phenomena were explored. 1. Typical ODE with variable coefficient problems reveals that both FEM exhibits the non comparable with exact solution. But HWCM gives the accurate solution as compared to exact. 2. As far as the singular valued ODEs are concerned, FEM is not comparable with exact solution but HWC method gives comparable solution with true solution. 3. In case of non-linear ODE, HWCM gives excellent solutions than FEM as compared with exact solutions, which is justified.

REFERENCES

[1] Baker, A. J 1995 Finite Element Computational Fluid Mechanics, Student Edition, McGraw- Hill Book Company, New York.
[2] Bathe, K. J 1982 Finite Procedures in Engineering Analysis, Prentice Hall, Englewood Cliffs, N. J.
[3] Rao SS 1989 The Finite Element Methods in Engineering, Second Edition, Pergamum Press, New York.
[4] Reddy J. N 1993 An Introduction to Finite Element Method, Second Edition, McGraw- Hill Book Company.
[5] Segerlind LJ 1984 Applied Finite Element Analysis. Second Edition, John Wiley and Sons.
[6] Zienkiewicz O. C and Taylor RL 2000 The Finite Element Method, Vol1, The Basis, Butterworth and Heinemann, London.
[7] Lepik U, 2006."Numerical solution of differential equations using Haar wavelets'' Mathematics and Computers in simulation (Elsevier), 68, pp.127-143,
[8] Lepik, U 2006. ''Numerical Solution of evolution equations by the Haar wavelet Method,'' Applied Mathematics and Computation (Elsevier), 185, pp.695-704
[9] Siraj-ul-Islam, Imran Aziz, Bozidar Sarlet 2010, The numerical solution of Second-order Boundary–value problems by collocation method with the Haar wavelets, Mathematical and Computer Modeling 52, pp 1577-1590.
[10] Goswami, J. C, Chan, 1989.Fundamentals of Wavelets. Theory, Algorithms and applications, John Wiley and Sons. New York.

Table1. Comparison of FEM and HWCM with Exact solutions for N=16 of the Test Problem 1.

$t(=1/32)$	FEM (F)	Exact (E)	HWCM (H)	Absolute Errors	
				$\|E-F\|$	$\|E-H\|$
1	0.0461	0.0848	0.0343	0.0387	0.0504
3	0.1377	0.2526	0.1241	0.0772	0.1284
5	0.2278	0.4144	0.2458	0.1149	0.1686
7	0.3158	0.5668	0.3977	0.1515	0.1690
9	0.4010	0.7063	0.5683	0.1866	0.1380
11	0.4828	0.8302	0.7370	0.2199	0.0932
13	0.5605	0.9362	0.8827	0.2510	0.0535
15	0.6335	1.0228	0.9921	0.2795	0.0306
17	0.7012	1.0889	1.0634	0.3053	0.0255
19	0.7633	1.1344	1.1023	0.3280	0.0321
21	0.8191	1.1596	1.1172	0.3474	0.0424
23	0.8684	1.1655	1.1156	0.3634	0.0499
25	0.9107	1.1534	1.1025	0.3758	0.0509
27	0.9457	1.1254	1.0812	0.3844	0.0442
29	0.9732	1.0835	1.0531	0.3893	0.0304
31	0.9930	1.0302	1.0191	0.3904	0.0110

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

Table 2. Error analysis of the Test Problem 1.

N	L_∞ (FEM)	MRE(FEM)	RPD(FEM)	L_∞ (HWCM)	MRE(HWCM)	RPD(HWCM)
16	0.3904	4.6010	8.0101	0.1690	1.9922	0.3210
32	0.3956	9.3177	8.1550	0.1728	4.0702	0.3210
64	0.3983	18.7588	8.2276	0.1737	8.1832	0.3210
128	0.3996	37.6375	8.2639	0.1740	16.3873	0.3210
256	0.4002	75.3957	8.2820	0.1740	32.7844	0.3210

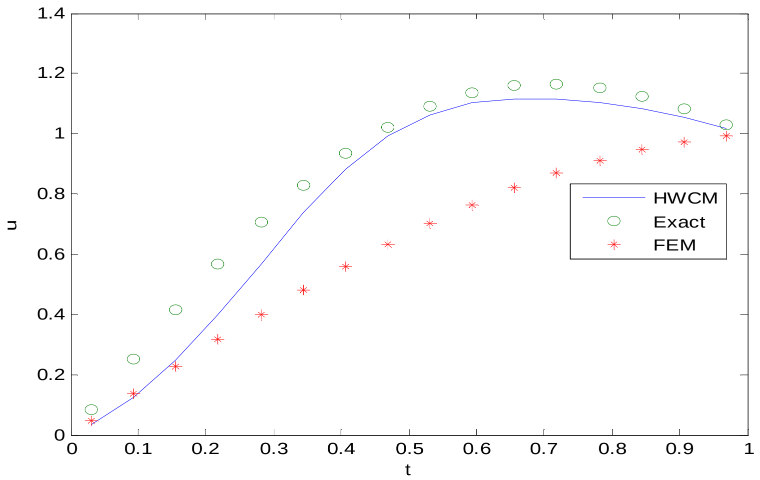

Fig. 1. Comparison of FEM & HWCM solution with exact solution for N=32 of Test Problem 1.

Table 3. Comparison of FEM and HWCM with Exact solutions for N=16 of the Test Problem 2.

$t(=1/32)$	FEM (F) 1.0e+03 *	Exact (E)	HWCM (H)	Absolute Errors $\|E-F\|$ 1.0e+03 *	Absolute Errors $\|E-H\|$ 1.0e-03 *
1	5.6691	1.0010	1.0010	5.6681	0.0011
3	4.4235	1.0088	1.0088	4.4225	0.0014
5	4.2962	1.0247	1.0247	4.2952	0.0063
7	4.2933	1.0490	1.0490	4.2922	0.0137
9	4.3041	1.0823	1.0823	4.3031	0.0233
11	4.3166	1.1254	1.1254	4.3155	0.0350
13	4.3292	1.1794	1.1794	4.3280	0.0485
15	4.3419	1.2457	1.2457	4.3407	0.0634
17	4.3547	1.3261	1.3260	4.3533	0.0790
19	4.3674	1.4227	1.4226	4.3660	0.0942
21	4.3802	1.5383	1.5382	4.3787	0.1075
23	4.3931	1.6763	1.6762	4.3914	0.1167
25	4.4060	1.8411	1.8410	4.4041	0.1186
27	4.4189	2.0379	2.0378	4.4169	0.1083
29	4.4319	2.2734	2.2733	4.4296	0.0789
31	4.4449	2.5561	2.5561	4.4423	0.0206

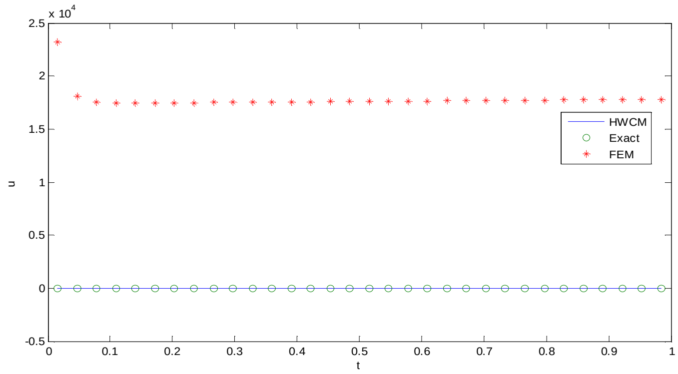

Fig. 2. Comparison of FEM & HWCM solutions with exact solution for N=32 of Test Problem 2

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

Table 2. Error analysis of the Test Problem 2.

N	FEM			HWCM		
	L_∞	L_{mre}	L_{rpd}	L_∞	L_{mre}	L_{rpd}
8	1.3534e+03	1.3481e+03	3.0328e+07	3.7226e-04	3.7081e-04	1.5843e-06
16	5.6681e+03	5.6625e+03	5.2565e+08	1.1858e-04	1.1846e-04	1.8227e-07
32	2.3204e+04	2.3199e+04	8.7575e+09	3.1412e-05	3.1404e-05	1.3156e-08
64	9.3906e+04	9.3900e+04	1.4300e+11	7.9072e-06	7.9067e-06	8.3507e-10
128	3.7782e+05	3.7782e+05	2.3114e+12	1.9978e-06	1.9977e-06	5.3740e-11
256	1.5157e+06	1.5157e+06	3.7172e+13	4.9990e-07	4.9990e-07	3.3662e-12

Table 5. Comparison of FEM and HWCM with Exact solutions for N=16 of the Test Problem 3.

$t(=1/32)$	FEM (F)1.0e+04 *	Exact (E)	HWCM (H)	Absolute Errors $\|E-F\|$ 1.0e+04 *	Absolute Errors $\|E-H\|$ 1.0e-03 *
1	-3.301119	0.001007	0.001037	3.301119	0.030512
3	-2.566184	0.009613	0.009558	2.199910	0.054951
5	-2.483446	0.028228	0.028095	2.566185	0.133286
7	-2.473179	0.058319	0.058119	2.443285	0.199695
9	-2.470965	0.101348	0.101085	2.483449	0.262939
11	-2.469647	0.158782	0.158458	2.469257	0.324493
13	-2.468428	0.2320861	0.231701	2.473185	0.384798
15	-2.467222	0.322723	0.322279	2.471073	0.443985
17	-2.466017	0.432159	0.431657	2.470975	0.502071
19	-2.464812	0.561859	0.561300	2.470207	0.559018
21	-2.463609	0.713287	0.712672	2.470975	0.614765
23	-2.462406	0.887908	0.887239	2.470207	0.669237
25	-2.461203	1.087188	1.086466	2.469663	0.722352
27	-2.460001	1.312591	1.311817	2.469044	0.774025
29	-2.458800	1.565582	1.564758	2.468451	0.824176
31	-2.457600	1.847625	1.846753	2.457199	0.872718

Table 6. Error analysis of the Test Problem3.

N	L_∞ (FEM)	MRE(FEM)	RPD(FEM)	L_∞ (HWCM)	MRE(HWCM)	RPD(HWCM)
16	3.3011e+04	3.2779e+07	2.5893e+10	8.7271e-04	0.8665	2.8786e-05
32	1.3157e+05	5.3063e+08	4.2927e+11	2.2143e-04	0.8930	1.7937e-06
64	5.2530e+05	8.5398e+09	6.9955e+12	5.5747e-05	0.9062	1.1202e-07
128	2.0992e+06	1.3704e+11	1.1298e+14	1.3983-e05	0.9128	7.0013e-09
256	8.3927e+06	2.1958e+12	1.8161e+15	3.4967e-06	0.9148	4.3675e-10

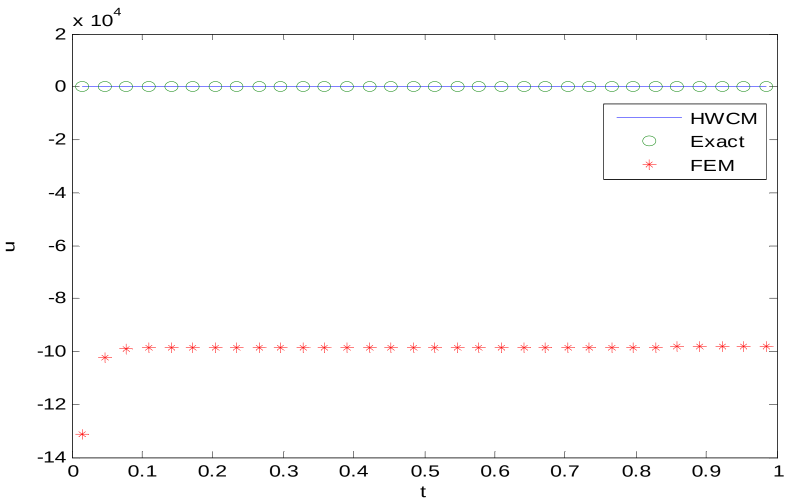

Fig. 3. Comparison of FEM & HWCM solution with exact solution for N=32 of Test Problem 3.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

Table 7. Comparison of HWCM and FEM with Exact solutions for N=16 of the Test Problem 4.

$t(=1/32)$	FEM (F) 1.0e+03 *	Exact (E)	HWCM (H)	Absolute Errors $\lvert E-F \rvert$ 1.0e+03 *	$\lvert E-H \rvert$ 1.0e-04 *
1	3.9125	0.9998	0.9998	3.9115	0.0009
3	3.0417	0.9985	0.9985	3.0407	0.0012
5	2.9436	0.9960	0.9960	2.9426	0.0056
7	2.9315	0.9921	0.9921	2.9305	0.0121
9	2.9288	0.9871	0.9871	2.9279	0.0208
11	2.9273	0.9809	0.9809	2.9263	0.0318
13	2.9258	0.9736	0.9736	2.9249	0.0450
15	2.9244	0.9653	0.9653	2.9234	0.0603
17	2.9230	0.9560	0.9560	2.9220	0.0778
19	2.9215	0.9460	0.9460	2.9206	0.0972
21	2.9201	0.9351	0.9351	2.9192	0.1184
23	2.9187	0.9236	0.9236	2.9178	0.1412
25	2.9173	0.9116	0.9115	2.9164	0.1653
27	2.9158	0.8990	0.8990	2.9149	0.1903
29	2.9144	0.8860	0.8860	2.9135	0.2161
31	2.9130	0.8728	0.8727	2.9121	0.2423

Table 8. Error analysis of the Test Problem 4.

N	FEM L_∞	FEM L_{mre}	FEM L_{rpd}	HWCM L_∞	HWCM L_{mre}	HWCM L_{rpd}
8	9.8412e+02	1.1364e+03	6.1882e+07	9.2374e-05	1.0504e-04	2.2181e-07
16	3.9115e+03	4.5166e+03	9.5948e+08	2.4231e-05	2.7763e-05	1.3842e-08
32	1.5591e+04	1.8003e+04	1.5104e+10	6.2101e-06	7.1429e-06	8.6492e-10
64	6.2248e+04	7.1877e+04	2.3968e+11	1.5723e-06	1.8120e-06	5.4055e-11
128	2.4875e+05	2.8724e+05	3.8189e+12	3.9558e-07	4.5633e-07	3.3784e-12
256	9.9453e+05	1.1484e+06	6.0974e+13	9.9210e-08	1.1450e-07	2.1115e-13

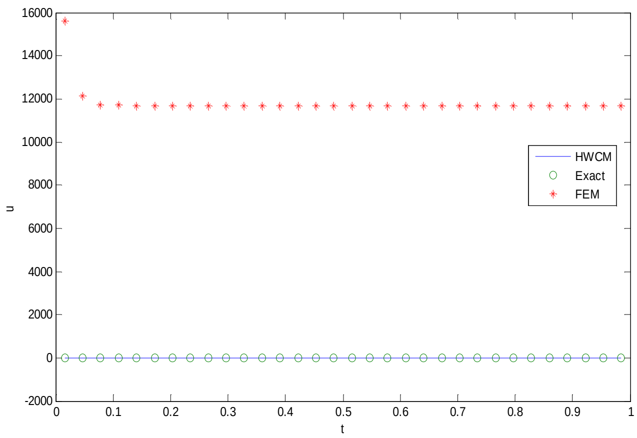

Fig. 4. Comparison of FEM & HWCM solution with exact solution for N=32 of Test Problem 4.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

Dr. S. C. Shiralashetti, was born in 1976. He received M.Sc., M.Phil, PGDCA, Ph.D. degree, in Mathematics from Karnatak University, Dharwad. He joined as a Lecturer in Mathematics in S. D. M. College of Engineering and Technology, Dharwad in 2000. Worked as a Assistant professor in Mathematics in Karnatak College Dharwad in 2009. From 2013 onwards working as a Associate Professor in the P.G. Department of studies in Mathematics, Karnatak University Dharwad. He has attended and presented More than 35 research articles in National and International Journals. He has published More than 36 research articles in National and International Journals and procedings. Area of Research: Numerical Analysis, Wavelet Analysis, CFD, Differential Equations, Integral Equations, Integro-Differential Eqns. H-index: 06; Citation index: 80; Ph.D. Working: 08; Research Projects Completed: 01: UGC-MRP (PI) (2013); Research Project Applied: UGC-MRP-01(2014); Life Member of the Association/Academy/Parishat/Society: 03; Awards: 04; Special Lectures Delivered: 20; Conference/Workshop organized as an organizing secretary / coordinator / Member: 10; Chaired the session in the National and International Conferences: 10; Conference/ Workshop/ Orientation/Refresher course attended: 23; Administrative Assignments completed: 02; Service given to the University in different capacities: 07.

Mr. P. B. Mutalik Desai, received his M.Sc., degree in Mathematics from Karnataka University Dharwad. He is pursuing his Ph.D degree in Dept of Engineering Mathematics from Visvesvaraya Technological University Belgaum in the field of Numerical Methods to solve Differential Equations.

His area of Interest includes Operations Research, Wavelets and Finite Element Method. Presently He is working as Associate Professor in KLE'S College of Engineering and Technology,Chikodi of Karnataka State.

Mr. A. B. Deshi, received his M.Sc., degree in Mathematics (2012) from Karnataka University Dharwad. He is pursuing his Ph.D degree in Dept of Mathematics,from Karnataka University Dharwad in the field of Numerical Methods to solve Differential Equations. His area of Interest includes Wavelets and Finite Element Method.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.