Fuzzy i.$\vee_f$-sets and fuzzy i.$\wedge_f$-sets

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Abstract. Recently, El-Naschie has shown that the notion of fuzzy topology may be relevant to quantum particle physics in connection with string theory and E-infinity space time theory. In this paper, we define $\wedge_f$-sets and $\vee_f$-sets in fuzzy ideal topological spaces and discuss their properties. Also define I.$\wedge_f$-sets and I. $\vee_f$-sets in fuzzy ideal topological spaces and discuss their properties.

Keywords- El-Naschie, E-infinity, topological.

I. INTRODUCTION

Fuzziness is one of the important and useful concepts in the modern scientific studies. This is because of the fact that since Zadeh first introduced the notion of fuzzy sets applications of this idea was made by many authors. Throughout the development of fuzzy sets, theory many interesting phenomena have been observed.

The topic of an ideal topological spaces was studied intensively by several authors [2, 7]. In [8], Mahmoud and in [9], Sarkar independently presented some of the ideal concepts in the fuzzy trend and studied many of their properties. The concept of fuzzy topology may be relevant to quantum particle physics particularly in connection with strong theory and E-infinity theory [3, 4, 5, 6]. In this paper, we define $\wedge_f$-sets and $\vee_f$-sets in fuzzy ideal topological spaces and discuss their properties. Also define I.$\wedge_f$-sets and I.$\vee_f$-sets in fuzzy ideal topological spaces and discuss their properties.

II. PRELIMINARIES

Throughout this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by A $\leq$ X, then is characterized by a membership function in the sense of Zadeh [10]. The basic fuzzy sets are the empty set, the whole set and the class of all fuzzy subsets of X which will be denoted by 0, 1 and I^X, respectively. A subfamily τ of I^X is called fuzzy topology due to Chang [1]. By (X, τ) or X, we mean a fuzzy topological space in Chang's sense. A fuzzy point in X with support x $\in$ X and value $\alpha(0 < \alpha \leq 1)$ is denoted by x_α.

For a fuzzy set A in X, cl(A), int(A) and 1 − A will, respectively, denote the closure, interiorand complement of A. A nonempty collection of fuzzy sets I of a set X is called a fuzzy ideal [8] if and only if (1) if A $\in$I and A $\leq$ B, then B $\in$I, (2) if A $\in$I and B $\in$I, then A $\vee$ B $\in$ I.

The triple (X, τ, I) means a fuzzy topological space with a fuzzy ideal I and fuzzy topology τ. For (X, τ, I), the fuzzy local function of A $\leq$ X with respect to τ and I denoted by $A^?(\tau, I)$ (briefly $A^?$) and is defined $A^?(\tau, I)$

= $\vee\{x \in X : A \wedge U \in I/$ for every $U \in \tau\}$. While $A^?$ is the union of the fuzzy points such that if U $\in$ τ and E $\in$I, then there is at least one y $\in$ X for which $U(y) + A(y) - 1 > E(y)$. Fuzzy closure operator of a fuzzy set A in (X, τ, I) is defined as $cl^?(A) = A \vee A^?$. In (X, τ, I), the collection $\tau^?(I)$ means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class β = {U − E : U $\in$ τ, E $\in$ I} as a base. This topology is considered as generalization of the ordinary one.

Definition 2.1. *A fuzzy subset A of a fuzzy topological space (X, τ, I) is said to be fuzzy ?-closed if $A^? \leq A$. The complement of fuzzy ?-closed set is fuzzy ?-open.*

III. FUZZY $\wedge$-SETS AND FUZZY $\vee$-SETS

Definition 3.1. *Let A be a fuzzy subset of a fuzzy topological space (X, τ). We define the subsets $A_f^\wedge$ and $A_f^\vee$ as follows:*

(1) $A_f^\wedge = \vee\{U : A \leq U$ *and U is fuzzy open*$\}$.

(2) $A^\vee_f = \wedge\{F : F \leq A$ *and F is fuzzy closed*$\}$.

Lemma 3.2. *For fuzzy subsets A, B and A_i, i$\in$ Δ, of a fuzzy topological space (X, τ) the following properties hold:*

(1) $A \leq A^\wedge_f$.

(2) $A \leq B \Rightarrow A^\wedge_f \leq B_f^\wedge$.

(3) $(A\wedge f)\wedge f = A\wedge f$.

(4) *If A is fuzzy open then* $A = A^\wedge_f$.

(5) $\vee\{(A_i)^\wedge_f : i\in\Delta\} = (\vee\{A_i : i \in \Delta\})_f^\wedge$. (6) $(\vee\{A_i : i \in \Delta\})^\wedge_f \leq \wedge\{(A_i)^\wedge_f : i\in\Delta\}$.

(7) $(1 - A)^\wedge_f = 1 - A^\vee_f$.

Proof. (1), (2), (4) and (6) are immediate consequences of Definition 3.1.

(3) From (1) and (2) we have $A^\wedge_f \leq (A^\wedge_f)^\wedge_f$. If $x_\lambda \in/ A^\wedge_f$, then there exists a fuzzy open set U such that A$\leq$U and $x_\lambda \in/$U. Hence $A_f^\wedge \leq$U by Definition 3.1 and so $x \in/ (A^\wedge_f)^\wedge_f$. Thus $(A\wedge f)\wedge f \leq A\wedge f$. Hence $(A\wedge f)\wedge f = A\wedge f$.

(5) Let A = $\vee\{A_i : i\in\Delta\}$. Therefore $A_i \leq \vee\{A_i : i \in \Delta\}$. By (2) $(A_i)^\wedge_f \leq (\vee\{A_i : i \in \Delta\})_f^\wedge$. Hence $\vee\{(A_i)^\wedge_f : i\in\Delta\} \leq (\vee\{A_i : i \in \Delta\})_f$. Suppose $x_\lambda \notin \vee\{(A_i)_f^\wedge : i \in \Delta\}$, then for each i$\in$ Δ, there exists a fuzzy open set U_i such that $A_i \leq U_i$ and $x_\lambda \in/ U_i$. If U = $\vee\{U_i : i \in \Delta\}$ then U is fuzzy open set with A $\leq$ U and $x_\lambda \in/$ U. Therefore $x_\lambda \in/ A^\wedge_f$. Hence $(\vee\{A_i : i \in \Delta\})^\wedge_f \leq \vee\{(A_i)_f^\wedge : i \in \Delta\}$.

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(7) $(1 - A)^{\wedge}_{f} = \wedge\{U : 1 - A \le U$ and U is fuzzy open$\}$
$= 1 - \vee \{1 - U : 1 - U \le A$ and $1 - U$ is fuzzy closed$\}$
$= 1 - A^{\vee}_{f}$.

Remark 3.3. *In Lemma 3.2, the equality in (6) does not hold as per the following example.*

Example 3.4. *Let X={a, b, c} and A, B be fuzzy sets of X defined as follows : A(a) = 0.3, A(b) = 0.6, A(c) = 0.9, B(a) = 0.2, B(b) = 0.7, B(c) = 0.9, C(a) = 0.2, C(b) = 0.6, C(c) = 0.9. We put τ = {0, C, 1} and* I *= {0}.*

Then $A^{\wedge}_{f} = 1$, $B^{\wedge}_{f} = 1$ *and* $(A \wedge B)^{\wedge}_{f} = C$.
Therefore $(A \wedge B)^{\wedge}_{f} = C \lneq 1 = A^{\wedge}_{f} \wedge B^{\wedge}_{f}$.

Lemma 3.5. *For subsets A, B and A_i, i∈ Δ, of a fuzzy*

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topological space (X, τ) the following properties hold:

(1) $A^{\vee}_{f} \le A$
(2) $A \le B \Rightarrow A^{\vee}_{f} \le B^{\vee}_{f}$.
(3) $(A^{\vee}_{f})^{\vee}_{f} = A^{\vee}_{f}$.
(4) *If A is fuzzy closed then* $A = A^{\vee}_{f}$.
(5) $(\wedge\{A_i : i \in \Delta\})^{\vee}_{f} = \wedge\{(A_i)^{\vee}_{f} : i\in \Delta\}$. (6) $\vee\{(A_i)^{\vee}_{f} : i\in \Delta\} \le (\vee\{A_i : i \in \Delta\})^{\vee}_{f}$.

Proof. (1), (2), (4) and (6) are immediate consequences of Definition 3.1.

(3) From (1) and (2) we have $(A^{\vee}_{f})^{\vee}_{f} \le A^{\vee}_{f}$. If $x_\lambda \in A^{\vee}_{f}$ then for some fuzzy closed set F≤A and $x_\lambda \in F$. Then F≤ $A^{\vee}_{f}$ by Definition 3.1.

Since F is fuzzy closed, again by Definition 3.1, $x_\lambda \in (A^{\vee}_{f})^{\vee}_{f}$.

(5) Let A=$\wedge\{A_i : i\in \Delta\}$. By (2) we have $(\wedge\{A_i : i \in \Delta\})^{\vee}_{f} < \wedge\{(A_i)^{\vee}_{f} : i\in \Delta\}$. If $x_\lambda \in \wedge\{(A_i)^{\vee}_{f} : i\in \Delta\}$, then for each i∈ Δ, there exists a fuzzy closed set F_i such that $F_i \le A_i$ and $x_\lambda \in F_i$. If F = $\wedge\{F_i : i \in \Delta\}$ then F is fuzzy closed with F≤A and $x_\lambda \in$F. Therefore $x_\lambda \in A^{\vee}_{f}$. Hence $\wedge\{(A_i)^{\vee}_{f} : i\in \Delta\} \le (\wedge\{A_i : i \in \Delta\})^{\vee}_{f}$.

Remark 3.6. *In Lemma 3.5, the equality in (6) does not hold. It is shown in the following example.*

Example 3.7. *Let X={a, b, c} and A, B be fuzzy sets of X defined as follows : A(a) = 0.3, A(b) = 0.6, A(c) = 0.9, B(a) = 0.2, B(b) = 0.7, B(c) = 0.9, C(a) = 0.3, C(b) = 0.7, C(c) = 0.9, D(a) = 0.7, D(b) = 0.3, D(c) = 0.1. We put τ = {0, D, 1} and* I *= {0}. Then* $A^{\vee}_{f} = 0$,
$B^{\vee}_{f} = 0$ *and* $(A \vee B)^{\wedge}_{f} = C$. *Therefore* $A^{\vee}_{f} \vee B^{\vee}_{f} = 0 \lneq C = (A \vee B)^{\wedge}_{f}$.

Definition 3.8. *A fuzzy subset A of a fuzzy topological space (X, τ) is said to be a*

(1) $\wedge_f$*-set if* $A = A^{\wedge}_{f}$.
(2) $\vee_f$*-set if* $A = A^{\vee}_{f}$.

Remark 3.9. *0 and 1 are always $\wedge_f$-sets and $\vee_f$-sets.*

Theorem 3.10. *Let (X, τ) be a fuzzy topological space. Then the following hold.*

(1) *Arbitrary union of $\wedge_f$-sets is a $\wedge_f$-set.*
(2) *Arbitrary intersection of $\vee_f$-sets is a $\vee_f$-set.*

Proof. (1) Let $\{A_i : i \in \Delta\}$ be a family of $\wedge_f$sets. If A=$\vee\{A_i : i \in \Delta\}$, then by Lemma 3.2, $A^{\wedge}_{f} = \vee\{(A_i)^{\wedge}_{f} : i \in \Delta\} = \vee\{A_i : i \in \Delta\}$ = A. Hence A is a $\wedge_f$-set.

(2) Let $\{A_i : i \in \Delta\}$ be a family of $\vee_f$-sets. If A=$\wedge\{A_i : i \in \Delta\}$, then by Lemma 3.5, $A^{\vee}_{f} = \wedge\{(A_i)^{\vee}_{f} : i \in \Delta\} = \wedge\{A_i : i \in \Delta\}$ = A. Hence A is a $\vee_f$-set.

Theorem 3.11. *Let (X, τ) be a fuzzy topological space. Then the following hold.*

(1) *Arbitrary intersection of $\wedge_f$-sets is a $\wedge_f$-set.*
(2) *Arbitrary union of $\vee_f$-sets is a $\vee_f$-set.*

Proof. (1) Let $\{A_i : i \in \Delta\}$ be a family of $\wedge_f$sets. If A = $\wedge\{A_i : i \in \Delta\}$, then by Lemma 3.2, $A^{\wedge}_{f} \le \wedge\{(A_i)^{\wedge}_{f} : i \in \Delta\} = \wedge\{A_i : i \in \Delta\}$ = A. Again by Lemma 3.2, A≤ $A^{\wedge}_{f}$. Hence A is a $\wedge_f$-set.

(2) Let $\{A_i : i \in \Delta\}$ be a family of $\vee_f$-sets. If A = $\vee\{A_i : i \in \Delta\}$, then by Lemma 3.5, $A^{\vee}_{f} \ge \vee\{(A_i)^{\vee}_{f} : i \in \Delta\} = \vee\{A_i : i \in \Delta\}$ = A. Again by Lemma 3.5, $A^{\vee}_{f} \le$ A. Hence A is a $\vee_f$-set.

IV. GENERALIZED ΛF-SETS AND VF-SETS IN FUZZY IDEAL TOPOLOGICAL SPACES

Definition 4.1. *A fuzzy subset A of a fuzzy ideal topological space (X, τ,* I*) is said to be*

(1) *fuzzy* I.$\wedge_f$*-set if* $A^{\wedge}_{f} \le F$ *whenever A≤F and F is fuzzy ?-closed.*
(2) *fuzzy* I.$\vee_f$*-set if 1−A is a fuzzy* I.$\wedge_f$*set.*

Proposition 4.2. *Let (X, τ,* I*) be a fuzzy ideal topological space. Then the following hold:*

(1) *Every $\wedge_f$-set is a fuzzy* I.$\wedge_f$*-set but not conversely.*
(2) *Every $\vee_f$-set is a fuzzy* I.$\vee_f$*-set but not conversely.*

Proposition 4.3. *Every fuzzy open set is fuzzy* I.$\wedge_f$*-set but not conversely.*

Proof. Let A≤F where F is fuzzy ?-closed. If A is fuzzy open, then $A^{\wedge}_{f}$ = A≤F. Hence A is fuzzy I.$\wedge_f$-set.

Example 4.4. *Let X={a, b, c} and A be a fuzzy set of X defined as follows : A(a) = 0.3, A(b) = 0.6, A(c) = 0.9. We put τ = {0, 1} and* I *= {0}. Then fuzzy ?-closed sets are 0 and 1. Therefore A is fuzzy* I.$\wedge_f$*-set but not $\wedge_f$-set and A^c is fuzzy* I.$\vee_f$*-set but not $\vee_f$-set.*

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Theorem 4.5. *A fuzzy subset A of a fuzzy ideal topological space (X, τ,* I*) is a fuzzy* I.$\vee_f$*-set if and only if U≤* $A^{\vee}_f$ *whenever U≤A and U is fuzzy ?-open.*

Proof. Suppose that A is a fuzzy I.$\vee_f$-set of X and U is a fuzzy ?-open set such that U≤A. Then 1−A≤1−U and 1−U is fuzzy ?-closed.
Since 1−A is a fuzzy I.$\wedge_f$-set, we have $(1-A)^{\wedge}_f \leq 1-$U and so $1-A^{\vee}_f \leq 1-$U, by Lemma 3.2. Therefore, U≤ $A^{\vee}_f$.

Conversely, assume that U≤ $A^{\vee}_f$ whenever U≤A and U is fuzzy ?-open. Suppose 1−A≤F and F is fuzzy ?-closed. Then, 1−F≤A and 1−F is fuzzy ?-open. Therefore, 1−F≤$A^{\vee}_f$ and so $1-A^{\vee}_f\leq$F. By Lemma 3.2, we have $(1-A)^{\wedge}_f \leq$F. Hence 1−A is a fuzzy I.$\wedge_f$-set and so A is a fuzzy I.$\vee_f$-set

Theorem 4.6. *Let A be a fuzzy subset of a fuzzy ideal topological space (X, τ,* I*) such that* $A^{\vee}_f$ *is a fuzzy ?-closed set. If F = 1, whenever F is fuzzy ?-closed and* $A^{\vee}_f \wedge(1-A)\leq F$*, then A is a fuzzy* I.$\vee_f$*-set*

Proof. Let U be a fuzzy ?-open set such that U≤A. Since $A^{\vee}_f$ is fuzzy ?-closed, $A^{\vee}_f \wedge(1-U)$ is fuzzy ?-closed. By hypothesis, $A^{\vee}_f \vee(1-U) = 1$.
This implies that U≤ $A^{\vee}_f$. Hence A is a fuzzy
I.$\vee_f$-set.

The set of all fuzzy I.$\vee_f$-sets is denoted by $D^{\vee}_{f\mathcal{I}}$ and the set of all fuzzy I.$\vee_f$-sets by $D^{\wedge}_{f\mathcal{I}}$

Definition 4.7. *Let (X, τ,* I*) be a fuzzy ideal topological space and A be a fuzzy subset of X. Then f-*$cl_I^{\wedge}$*(A) =∧{U : A≤U and U∈*$D_f^{\wedge}{}_I$*} and f-*$int^{\vee}_I$*(A)=∨{F : F≤A and F∈*$D_f^{\vee}{}_I$*}.*

Theorem 4.8. *Let (X, τ,* I*) be a fuzzy ideal topological space and* A_i*, i∈ Δ be fuzzy subsets of X. Then the following hold.*

(1) *If* $\{A_i\ i \in D^{\wedge}_{f\mathcal{I}}\}\in D^{\wedge}_{fI}$ *for all i∈ Δ, then ∨{*A_i *: i ∈ Δ}∈* D_{fI}*.*

(2) *If* $A_i \in D_f^{\vee}{}_I$ *for all i∈ Δ, then ∧{*A_i *: i ∈ Δ}∈* $D_f^{\vee}{}_I$*.*

Proof. (1) Let $A^i \in D^{\wedge}_{f\mathcal{I}}$ for all i∈ Δ. Suppose ∨{A_i : *i* ∈ Δ}≤F and F is fuzzy ?-closed. Then A_i ≤F for all i∈ Δ. So $(A_i)^{\wedge}_f$≤F for all i∈ Δ.
Therefore, $\vee\{(A_i)^{\wedge}_f\ :\ i\ \in\ \Delta\}\leq$F. By Lemma **3.2,** (∨{$A_i$: *i* ∈ Δ})$^{\wedge}_f$ = ∨{$(A_i)^{\wedge}_f$: *i* ∈ Δ}≤F. So ∨{A_i : *i* ∈ Δ}∈ $D_f^{\wedge}{}_I$.

(2) Let $A^i\ \in\ D^{\vee}_{f\mathcal{I}}$ for all i∈ Δ. Then, 1−A_i ∈ $D_f^{\wedge}{}_I$ for all i∈ Δ. So, by (1), ∨{1−A_i : *i* ∈ Δ}∈ $D_f^{\wedge}{}_I$. Hence 1−∧{A_i : *i* ∈ Δ}∈ $D_f^{\wedge}{}_I$ and so ∧{A_i : .

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Theorem 4.9. *Let (X, τ,* I*) be a fuzzy ideal topological space and A, B be fuzzy subset of X. Then f-*$cl_I^{\wedge}$ *is a kuratowski closure operator on X.*

Proof. (1) Since $0^{\wedge}_f$ = 0, 0 ∈ $D_f^{\wedge}{}_I$ and so f-$cl_I^{\wedge}$(0) = 0.

(2) From the definition of $f\text{-}cl^{\wedge}_{\mathcal{I}}$(A), it is clear that A≤*f*-$cl_I^{\wedge}$(A).

(3) We have {U : A∨B≤U, U∈ $D_f^{\wedge}{}_I$}≤{U : A≤U, U$\in D^{\wedge}_{f\mathcal{I}}\}$. So $f\text{-}cl^{\wedge}_{\mathcal{I}}(\text{A})\leq f\text{-}cl^{\wedge}_{\mathcal{I}}(\text{A}\vee\text{B})$. Similarly, *f*-$cl_I^{\wedge}$(B)≤*f*-$cl_I^{\wedge}$(A∨B). Therefore, *f*$cl_I^{\wedge}$(A)∨*f*-$cl_I^{\wedge}$(B)≤*f*-$cl_I^{\wedge}$(A∨B). On the other hand, if $x_\lambda \notin f\text{-}cl^{\wedge}_{\mathcal{I}}(\text{A})\vee f\text{-}cl^{\wedge}_{\mathcal{I}}$(B), then x_λ ∈/*f*$cl_I^{\wedge}$(A). So there exists U_1 ∈ $D_f^{\wedge}{}_I$ such that A≤U_1 but x_λ ∈/U_1. Similarly, there exists $U^2 \in D^{\wedge}_{f\mathcal{I}}$ such that B≤U_2 but x_λ ∈/U_2. Let U = U_1∨U_2. Then, by Theorem 4.8, U∈ $D_f^{\wedge}{}_I$ such that A∨B≤U but x_λ ∈/U. So x_λ ∈/*f*-$cl_I^{\wedge}$(A∨B). Therefore, *f*-$cl_I^{\wedge}$(A∨B) ≤ *f*-$cl_I^{\wedge}$(A)∨*f*-$cl_I^{\wedge}$(B) which implies that *f*-$cl_I^{\wedge}$(A∨B) = *f*-$cl_I^{\wedge}$(A)∨*f*$cl_I^{\wedge}$(B).

(4) Now {U : A≤U, U∈ $D_f^{\wedge}{}_I$} = {U : *f*$cl_I^{\wedge}$(A)≤U, U∈ $D_f^{\wedge}{}_I$} by the definition of *f*$cl_I^{\wedge}$ operator and so *f*-$cl_I^{\wedge}$(A) = *f*-$cl_I^{\wedge}$(*f*-$cl_I^{\wedge}$(A)). Hence *f*-$cl_I^{\wedge}$ is a kuratowski closure operator.

Definition 4.10. *Let (X, τ,* I*) be a fuzzy topological space and A be a fuzzy subset of X. Then A is said to be fuzzy* I_g*-closed (briefly f.*I_g*closed) set if* $A^?$ *≤U whenever A≤U and U is fuzzy open.*

The complement of f-I_g*-closed set is f-*I_g*open.*

Theorem 4.11. *Let (X, τ,* I*) be a fuzzy ideal topological space. Then 1−f-*$cl_I^{\wedge}$*(A) = f*$int^{\vee}_I$*(1 − A) for every fuzzy subset A of X.*

Proof. 1−*f*-$cl_I^{\wedge}$(A) = 1−∧{U : A≤U, U∈ $D_f^{\wedge}{}_I$}
= ∨{1−U : $1-\text{U}\leq 1-\text{A},\ 1-\text{U}\in D^{\vee}_{f\mathcal{I}}\}$ = *f* *int*$^{\vee}_I$(1−A).

Theorem 4.12. *A fuzzy subset A of a fuzzy ideal topological space (X, τ,* I*) is an f-*I_g*closed set if and only if* $cl^?(A)\leq A^{\wedge}_f$*.*

Proof. Suppose that A is a *f*-I_g-closed subset of X. Let x_λ ∈$cl^?$(A). Suppose $x_\lambda \notin A^{\wedge}_f$.
Then there exists a fuzzy open set U containing A such that x_λ ∈/U. Since A is a *f*-$I_g$closed set, A≤U and U is fuzzy open, $cl^?$(A)≤U and so x_λ ∈/$cl^?$(A), a contradiction. Therefore, $cl^?(A)\leq A^{\wedge}_f$.

Conversely, suppose $cl^{\star}(\text{A})\leq A^{\wedge}_f$. If A≤U and U is fuzzy open, then $A^{\wedge}_f \leq U_f^{\wedge}$ = U and so $cl^?(A)\leq A^{\wedge}_f \leq$U. Therefore, A is *f*-$I_g$-closed.

Corollary 4.13. *A fuzzy subset A of a fuzzy ideal topological space (X, τ,* I*) is f-*I_g*-open if and only if* $A^{\vee}_f\leq int^?(A)$*.*

Proof. A is *f*-I_g-open subset of X iff 1−A is *f*-I_g-closed if and only if $cl^?(1-A)\leq(1-A)^{\wedge}_f$ if and only if $1-int^?(A)\leq 1-A^{\vee}_f$ if and only if $A^{\vee}_f\leq int^?(A)$.

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Theorem 4.14. *Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy subset of X. If $A^{\wedge}_f$ is a f-I_g-closed set, then A is a f-I_g-closed set.*

Proof. Let $A^{\wedge}_f$ be f-I_g-closed set. By Theorem 4.12, $cl^{?}(A^{\wedge}_f) \leq (A^{\wedge}_f)^{\wedge}_f = A^{\wedge}_f$. Since $A \leq A^{\wedge}_f$ and so $cl^{?}(A) \leq cl^{?}(A^{\wedge}_f) \leq A^{\wedge}_f$. Again, by Theorem 4.12, A is f-I_g-closed.

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