

# A Novel Fuzzy Soft Theory on Compact Spaces for Particular Issues

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**Abstract**— The aim of this work is to study some properties related to fuzzy soft topological spaces particularly fuzzy soft boundary point, fuzzy soft compact space, fuzzy soft open base and fuzzy soft open sub-base.

**Keywords**— Fuzzy soft set, fuzzy soft topological space, fuzzy soft interior, fuzzy soft closure, fuzzy soft boundary point, fuzzy soft neighborhood, fuzzy soft compact space, fuzzy soft open base, fuzzy soft open subbase, fuzzy soft basic open cover, fuzzy soft sub basic cover fuzzy soft closed base, fuzzy soft closed subbase.

## I. INTRODUCTION

We are not able to solve some kind of problems in Medical Science, Sociology, Economics, Environment, Engineering etc. by using classical methods because of the uncertainty. To deal with these uncertainties some theories like Fuzzy sets, rough sets intuitionistic fuzzy sets were developed as mathematical tools. [6] As these theories also have their own difficulties Soft theory was introduced by Molodtsov in 1999.

[5] Fuzzy soft set, a combination of fuzzy set and soft set was first introduced by Maji in 2001. [10] Later Topological structure of fuzzy soft sets has been introduced by B Tanay, MB Kandemir in 2011.[2] In 2012 J.Mahanta and PK Das introduced fuzzy soft point and studied the concept of neighborhood of a fuzzy soft point in a fuzzy soft topological space. They studied fuzzy soft closure and fuzzy soft interior etc.[4] T Simsekler and S Yuksel too studied and proved some results on this theory. They defined fuzzy soft open sets, fuzzy soft closed sets, fuzzy soft Q- neighborhood etc in 2012. [8] TJ Neog, DK Sut and GC Hazarika have established some properties and propositions related to fuzzy soft topological spaces in 2012. [3] In 2013 S Atmaca and I Zorlutuna introduced the notion of soft quasi-coincidence for fuzzy soft sets and used this notion to characterize fundamental concepts of fuzzy soft topological spaces such as fuzzy soft closures, fuzzy soft bases and fuzzy soft continuity. Some basic properties are also presented.

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## II. PRELIMINARIES

Let  $U$  be an universal set and  $E$  be a collection of all possible parameters with respect to  $U$ , where parameters are the characteristics or properties of objects in  $U$ .

**Definition 2.1:** [9]. Let  $A \subset E$  and  $\rho(U)$  be the set of all fuzzy sets in  $U$ . Then the pair  $(FS, A)$  denoted by  $FS_A$  is called a fuzzy soft set over  $U$ , if  $FS: A \rightarrow \rho(U)$  is a function.

**Definition 2.2:** [9]. Two fuzzy soft sets  $FS_A$  and  $FS_B$  are said to be disjoint if  $FS(a) \cap FS(b) = \emptyset, \forall a \in A, b \in B$ .

**Definition 2.3:** [9]. A fuzzy soft set  $FS_A$  is said to be a subset of a fuzzy soft set  $FS_B$ , if  $A \subset B$  and  $FS_A(a) \leq FS_B(a) \quad \forall a \in A$ .

**Definition 2.4:** [9]. The union of two fuzzy soft sets  $FS_A$  and  $FS_B$  over a common universe  $U$  is the fuzzy soft set  $FS_{A \cup B}$  and  $\forall e \in A \cup B$ , we have

$$FS_{A \cup B}(e) = \begin{cases} S_A(e), & \text{if } e \in A - B; \\ S_B(e), & \text{e} \in B - A; \\ S_A(e) \cup S_B(e), & \text{e} \in B \cap A. \end{cases} \quad \text{and We}$$

write  $S_A \cup S_B = S_{A \cup B}$ .

**Definition 2.5:** [9]. The intersection of two fuzzy soft sets  $FS_A$  and  $FS_B$  over a common universe  $U$  is the soft set  $FS_{A \cap B}$  and  $\forall e \in A \cap B, FS_{A \cap B}(e) = FS_A(e) \cap FS_B(e)$ .

We write  $FS_A \cap FS_B = FS_{A \cap B}$ .

**Definition 2.6:** [4]. The complement of a fuzzy soft set  $FS_A$ , denoted by  $FS'_A$  and is defined by  $FS'_A(a) = (FS_A(a))' \forall a \in A$ .

**Definition 2.7:** [4]. A fuzzy soft set  $FS_A$  over  $U$  is called a fuzzy soft null set, denoted by  $FS_\emptyset$  if  $A = E$  and  $\forall a \in E, FS(e) = \bar{0}$ .

**Definition 2.8:** [4]. A fuzzy soft set  $FS_A$  over  $U$  is called a fuzzy soft fullset, denoted by  $\bar{S}_E$ , if  $A = E$  and  $\forall a \in E, FS(e) = \bar{1}$ .

**Definition 2.9:** [4]. Let  $FS_A$  be a fuzzy soft set,  $\{FS_{A\alpha}\}$  be the class of all fuzzy soft subsets of  $FS_A$  and  $\mathbf{T}$  be a subclass of  $\{FS_{A\alpha}\}$ . Then  $\mathbf{T}$  is called a fuzzy soft topology on  $FS_A$  if the following conditions hold.

(i).  $FS_\emptyset, \bar{S}_A \in \mathbf{T}$ ;

(ii).  $FS_A, FS_B \in \mathbf{T} \Rightarrow FS_A \cap FS_B \in \mathbf{T}$  ;

(iii).  $\{(FS_{A\lambda} | \lambda \in \Lambda) \subset \mathbf{T} \Rightarrow \bigcup_{\lambda \in \Lambda} FS_{A\lambda} \in \mathbf{T}$ .

Then  $(FS_A, \mathbf{T})$  is called a fuzzy soft topological space. Members of  $\mathbf{T}$  are called fuzzy soft open sets and their complements are called fuzzy soft closed sets.

**Definition 2.10:** [4]. Let  $(FS_A, \mathbf{T})$  be a fuzzy soft topological space and  $FS_B \in \{S_{A\alpha}\}$ . Then the fuzzy soft topology  $\mathbf{T}_B = \{FS_B \cap FS_A \mid FS_A \in \mathbf{T}\}$  is called fuzzy soft subspace and  $(FS_B, \mathbf{T}_B)$  is called fuzzy soft subspace of  $(FS_A, \mathbf{T})$ .

**Definition 2.11:** [4]. A fuzzy soft set  $FS_A$  is said to be a fuzzy soft point in  $(FS_A, \mathbf{T})$ , denoted by  $FS_{Ae}$ , if for the element  $e \in A, FS(e) \neq \emptyset$  and  $FS(e') = \emptyset, \forall e' \in A - \{e\}$ .

**Definition 2.12:** [4]. The complement of a fuzzy soft point  $FS_{Ae}$  is a fuzzy soft point  $(FS_{Ae})'$  such that  $FS_{Ae}'(e) = 1 - FS(e)$ , and  $FS_{Ae}'(e') = 1, \forall e' \in A - \{e\}$ .

**Definition 2.13:** [2]. A fuzzy soft point  $FS_{Ae}$  is said to be in a fuzzy soft topological space  $\{FS_\phi, \bar{S}_E, FS_B\}$ , denoted by  $FS_{Ae} \in FS_B$  if for the element  $e \in A, FS_{Ae}(e) \leq FS_B(e)$ .

Example:- Let  $U = \{a, b, c, d\}$   $E = \{e1, e2, e3, e4, e5\}$ ,  
 $A = \{e1, e3, e4\}$ ,  $B = \{e1, e2, e3, e4\}$   
 $FS_A = \{FS(e1) = \{a, b, c, d\}, FS(e3) = \{a, b, c, d\}, FS(e4) = \{a, b, c, d\}\}$ .

$FS_B = \{FS(e1) = \{a, b, c, d\}, FS(e2) = \{a, b, c, d\}, FS(e3) = \{a, b, c, d\}, FS(e4) = \{a, b, c, d\}\}$ .

Here  $FS_{Ae1}, FS_{Ae3}$ , are the fuzzy soft points of the fuzzy soft topology  $\{FS_\phi, \bar{S}_E, FS_B\}$  but  $FS_{Ae4}$  is not a fuzzy soft point of  $\{FS_\phi, \bar{S}_E, FS_B\}$ .

And  $(FS_{Ae3})' = \{a, b, c, d\}$ .

**Definition 2.14:** [4] Let  $(FS_A, \mathbf{T})$  be a fuzzy soft topological space over  $FS_A, FS_B$  be a fuzzy soft subset of  $FS_A$  and  $FS_{Ax}$  be a fuzzy point in  $(FS_A, \mathbf{T})$ , then  $FS_B$  is called a fuzzy soft neighborhood of  $FS_{Ax}$  if there exists a fuzzy soft open set  $FS_C$  such that  $FS_{Ax} \in FS_C \subset FS_B$ .

**Definition 2.15:** [2]. The fuzzy soft interior of a fuzzy soft set  $FS_A$  denoted by  $\mathbf{I}(FS_A)$  is defined by the union of all fuzzy soft open sub sets of  $FS_A$ .

$i.e = \bigcup_i \{FS_{Bi} \mid FS_{Bi} \text{ is fuzzy soft open sub set of } FS_A\}$

By the definition it is clear that

(i).  $\mathbf{I}(FS_A)$  is a fuzzy soft open set (ii).  $\mathbf{I}(FS_A) \subset FS_A$  and (iii). If  $FS_A$  is fuzzy soft open then  $FS_A = \mathbf{I}(FS_A)$ .

(iv).  $\mathbf{I}(FS_A)$  is the largest fuzzy soft open subset of  $FS_A$

**Definition 2.16:** [10]. The fuzzy soft closure of a fuzzy soft set  $FS_A$  denoted by  $\bar{S}_A$  is defined by the intersection of all fuzzy soft closed super sets of  $FS_A$ .

$i.e = \bigcap_i \{FS_{Bi} \mid FS_{Bi} \text{ is fuzzy soft closed super set of } FS_A\}$

By the definition it is clear that

(i).  $\bar{S}_A$  is a fuzzy soft closed set (ii).  $FS_A \subset \bar{S}_A$  and (iii). If  $FS_A$  is fuzzy soft closed then  $FS_A = \bar{S}_A$ .

**Example :-** Let  $U = \{a, b, c, d\}$   $E = \{e1, e2, e3, e4\}$ ,  $A = \{e1, e3, e4\}$ ,  $B = \{e1, e2, e3, e4\}$

$FS_A = \{S(e1) = \{a, b, c, d\}, FS(e2) = \{a, b, c, d\}, FS(e3) = \{a, b, c, d\}, FS(e4) = \{a, b, c, d\}\}$

$FS_B = \{S(e1) = \{a, b, c, d\}, FS(e2) = \{a, b, c, d\}, FS(e3) = \{a, b, c, d\}, FS(e4) = \{a, b, c, d\}\}$ .

Consider a fuzzy soft topology  $\mathbf{T} = \{FS_\phi, \bar{S}_E, FS_A, FS_B\}$

$(FS_A)' = \{FS'(e1) = \{a, b, c, d\}, FS'(e2) = \{a, b, c, d\}, FS'(e3) = \{a, b, c, d\}, FS'(e4) = \{a, b, c, d\}\}$

$(FS_B)' = \{FS'(e1) = \{a, b, c, d\}, FS'(e2) = \{a, b, c, d\}, FS'(e3) = \{a, b, c, d\}, FS'(e4) = \{a, b, c, d\}\}$ .

Clearly  $(FS_A)'$  and  $(FS_B)'$  are fuzzy soft closed sets

Now write  $FS_C = \{FS'(e1) = \{a, b, c, d\}, FS'(e2) = \{a, b, c, d\}, FS'(e3) = \{a, b, c, d\}, FS'(e4) = \{a, b, c, d\}\}$ .

Its clear that  $FS_C \subset (FS_B)'$  and the fuzzy soft closure of  $S_C$ ,

$\bar{S}_C = (FS_B)' \cap \bar{S}_E = (FS_B)' = \{FS'(e1) = \{a, b, c, d\}, FS'(e2) = \{a, b, c, d\}, FS'(e3) = \{a, b, c, d\}, FS'(e4) = \{a, b, c, d\}\}$ .

Here we can observe that  $FS_C \subset \bar{S}_C$

### III. FUZZY SOFT BOUNDARY POINT

**Definition 3.1 :** A fuzzy soft point  $FS_{Ae}$  is said to be a fuzzy soft boundary point of a fuzzy soft set  $FS_B$  if  $FS_{Ae} \in \bar{S}_B \cap \bar{S}_B'$

**Definition 3.2 :** The set of all fuzzy soft boundary points over a fuzzy soft set  $FS_A$  is called fuzzy soft boundary of the set  $FS_A$ . and is denoted by  $\mathbf{B}(FS_A)$ .

**Example:-** Take  $FS_A = \{FS'(e1) = \{a, b, c, d\}, FS'(e2) = \{a, b, c, d\}, FS'(e3) = \{a, b, c, d\}, FS'(e4) = \{a, b, c, d\}\}$

$FS_C = \{FS(e1) = \{a, b, c, d\}, FS(e2) = \{a, b, c, d\}, FS(e3) = \{a, b, c, d\}, FS(e4) = \{a, b, c, d\}\}$ .

$\bar{S}_C = \{FS(e1) = \{a, b, c, d\}, FS(e2) = \{a, b, c, d\}, FS(e3) = \{a, b, c, d\}, FS(e4) = \{a, b, c, d\}\}$ .

$(FS_C)' = \{FS'(e1) = \{a, b, c, d\}, FS'(e2) = \{a, b, c, d\}, FS'(e3) = \{a, b, c, d\}, FS'(e4) = \{a, b, c, d\}\}$ .

$$FS'(e3) = \{a_7, b_9, c_1, d_1\}, FS'(e4) = \{a_1, b_6, c_7, d_1\}.$$

$$\overline{S_C} = (FS_A)' \cap \overline{S_E} = (FS_A)' = \{ FS'(e1) = \{a_9, b_8, c_1, d_1\}, FS'(e2) = \{a_8, b_1, c_1, d_1\}, FS'(e3) = \{a_7, b_1, c_2, d_1\}, FS'(e4) = \{a_1, b_8, c_9, d_1\} \}$$

$$\text{Now } \mathbf{B}(FS_C) = \overline{FS_C \cap F \overline{S_C}} = \{FS(e1) = \{a_7, b_6, c_9, d_1\}, FS(e2) = \{a_7, b_1, c_6, d_6\}, FS(e3) = \{a_5, b_8, c_1, d_0\}, FS(e4) = \{a_9, b_5, c_4, d_0\}\}.$$

A fuzzy soft point,  $FS_A = \{ FS'(e1) = \{a_1, b_2, c_0, d_0\}, FS'(e2) = \{a_2, b_0, c_0, d_0\}, FS'(e3) = \{a_3, b_0, c_1, d_0\}, FS'(e4) = \{a_0, b_2, c_1, d_0\} \}$  is a fuzzy soft boundary point of  $FS_C$ . And we say  $FS_{Ae} \in \mathbf{B}(FS_C) \forall e \in A$ .

Note:- Being the intersection of two fuzzy soft closed sets, the fuzzy soft boundary set is fuzzy soft closed.

**Theorem 3.3:** Let  $(FS_A, T)$  be a fuzzy soft topological space and  $FS_C \in T$  then

- (i).  $\mathbf{B}(FS_C) \subset \overline{S_C}$  i.e. the fuzzy soft boundary of a fuzzy soft set is subset of the fuzzy soft boundary of the set.
- (ii).  $FS_C$  is fuzzy soft open set if and only if  $FS_C \cap \mathbf{B}(FS_C) = FS_\emptyset$
- (iii).  $FS_C$  is fuzzy soft closed if and only if  $\mathbf{B}(FS_C) \subset FS_C$

Proof:-

(i) By definition 3.2 we have  $\mathbf{B}(FS_C) = \overline{FS_C \cap F \overline{S_C}} \Rightarrow \mathbf{B}(FS_C) \subset F \overline{S_C}$ . (Also  $\mathbf{B}(FS_C) \subset \overline{S_C}$ )

(ii). Suppose  $FS_C$  is fuzzy soft open  $\Rightarrow (FS_C)'$  is fuzzy soft closed.

$$\therefore (FS_C)' = \overline{S_C} \Rightarrow \mathbf{B}(FS_C) \subset (FS_C)' \text{ Using (i)}$$

$\Rightarrow FS_C \cap \mathbf{B}(FS_C) = FS_\emptyset$   
Conversely suppose that

$$FS_C \cap \mathbf{B}(FS_C) = FS_\emptyset$$

$$FS_C \cap (F \overline{S_C} \cap \overline{S_C}) = FS_\emptyset \Rightarrow (FS_C \cap \overline{S_C}) = (FS_\emptyset)$$

$$\Rightarrow \overline{S_C} \subset (FS_C)' \Rightarrow$$

$(FS_C)' = \overline{S_C}$  (since  $(FS_C)' \subset F \overline{S_C}$ )  $\Rightarrow (FS_C)'$  is fuzzy soft closed set  $\Rightarrow FS_C$  is fuzzy soft open

(iii). Suppose  $FS_C$  is fuzzy soft closed, then  $FS_C = F \overline{S_C} \Rightarrow \mathbf{B}(FS_C) \subset FS_C$  using (i).

Conversely suppose that  $\mathbf{B}(FS_C) \subset FS_C$

$$\Rightarrow FS_C \cap \mathbf{B}(FS_C) = FS_\emptyset \Rightarrow FS_C \text{ is fuzzy soft open set}$$

(from (ii))

$\Rightarrow FS_C$  is fuzzy soft closed set.

#### IV. FUZZY SOFT COMPACTNESS

**Definition 4.1 :** Let  $(FS_A, T)$  be a fuzzy soft topological space. A class  $\{FS_{Gi}\}$  of fuzzy soft subsets of  $FS_A$  is said to be a fuzzy soft open cover of  $FS_A$  if each fuzzy soft point in  $FS_A$  belongs to at least one  $FS_{Gi}$ .

**Definition 4.2 :** A subclass of a fuzzy soft open cover which itself is an open cover is called a fuzzy soft subcover.

**Definition 4.3 :** A fuzzy soft compact space is a fuzzy soft topological space in which every fuzzy soft open cover has a finite fuzzy soft subcover.

**Definition 4.4 :** A fuzzy soft compact subspace of a fuzzy soft topological space is a fuzzy soft subspace which is fuzzy soft compact as a fuzzy soft topological space in its own right.

**Theorem 4.5:-** Any fuzzy soft closed subspace of a fuzzy soft compact space is fuzzy soft compact.

**Proof:-** Let  $(FS_A, T)$  be a fuzzy soft compact space.

Let  $FS_B$  be a fuzzy soft closed subspace of  $FS_A$ .

Now we have to prove that  $FS_B$  is fuzzy soft compact space.

For this we need to show that every fuzzy soft open cover of  $FS_B$  contains a finite fuzzy soft subcover.

Let  $\{FS_{Gi}\}$  be a fuzzy soft open cover of  $FS_B$ .

$$\text{By definition 4.1, } FS_B = \bigcup S_{Gi} \dots \dots \dots (a)$$

Since each  $FS_{Gi}$  is a fuzzy soft open set in  $FS_B, FS_{Gi} = FS_{Hi} \cap S_B \dots \dots \dots (b)$ , where  $FS_{Hi} \subset FS_A$  for each  $i$  by definition of relative fuzzy soft topology.

Since  $FS_B$  is fuzzy soft closed subspace of  $FS_A, FS_B'$  is fuzzy soft open subspace of  $FS_A$ .

$$\text{And it implies that } FS_A = FS_B \cup FS_B' \dots \dots \dots (c)$$

$$= \bigcup S_{Gi} \cup FS_B' \text{ (Using (a))}$$

$$= \bigcup S_{Gi} \cup S_B'$$

$$= \bigcup (FS_{Hi} \cap S_B) \cup S_B' \text{ (Using (b))}$$

$$= (\bigcup (FS_{Hi} \cap S_B) \cup FS_B \cup FS_B') \text{ By distributive property.}$$

$$= [(\bigcup (FS_{Hi} \cap S_B) \cup S_A)] \text{ (Using (c))}$$

$$= [\bigcup (FS_{Hi} \cap S_B) \cup S_B'] \text{, Since each } FS_{Hi} \text{ and } FS_B' \text{ are subsets of } FS_A.$$

$\{FS_{Hi}, FS_B'\}$  is a fuzzy soft cover of  $FS_A$ .

Since  $FS_A$  is fuzzy soft compact, this fuzzy soft open cover has a finite fuzzy soft subcover.

Let it be  $\{FS_{H1}, FS_{H2}, FS_{H3}, FS_{H4} \dots \dots \dots, FS_{Hn}, FS_B'\}$ .

$$\Rightarrow FS_A = FS_{H1} \cup FS_{H2} \cup FS_{H3} \cup FS_{H4} \dots \dots \dots \cup FS_{Hn} \cup FS_B' \dots \dots \dots (d)$$

Since  $FS_B \subset FS_A$  we have  $FS_B = FS_A \cap S_B$ .

$$\Rightarrow FS_B = [FS_{H1} \cup FS_{H2} \cup FS_{H3} \cup FS_{H4} \dots \dots \dots \cup FS_{Hn} \cup FS_B']$$

$FS_B$ . (Using (d)).

$$FS_B = [(FS_{H1} \quad S_B \quad (FS_{H2} \quad S_B) \dots \dots \dots (FS_{Hn} \quad S_B)] \quad (SF_B' \quad S_B).$$

$$FS_B = FS_{G1} \quad FS_{G2} \quad FS_{G3} \quad S_{G4} \dots \dots \dots FS_{Gn}$$

$$\Rightarrow FS_B = FS_{G1} \quad S_{G2} \quad FS_{G3} \quad S_{G4} \dots \dots \dots S_{Gn}.$$

$\{FS_{G1}, FS_{G2}, FS_{G3}, FS_{G4}, \dots, FS_{Gn}\}$  is a finite fuzzy soft subcover of  $FS_B$ .

Hence every fuzzy soft open cover of  $FS_B$  has a finite fuzzy soft subcover.

And hence  $FS_B$  is a fuzzy soft compact space.

Thus any fuzzy soft closed subspace of a fuzzy soft compact space is fuzzy soft compact. Hence the theorem is proved.

**Theorem 4.6:** A fuzzy soft topological space is fuzzy soft compact iff every class of fuzzy soft closed sets with empty intersection has a finite subclass with empty intersection.

**Proof:** This is a direct consequence of the fact that a class of fuzzy soft open sets is an open cover iff the class of all their complements has empty intersection.

**Definition 4.7:** A class of fuzzy soft subsets of a nonempty fuzzy soft set is said to have the finite intersection property if every finite subclass has non-empty intersection.

**Theorem 4.8:** A fuzzy soft topological space is fuzzy soft compact iff every class of fuzzy soft closed sets with the finite intersection property has non-empty intersection.

**Definition 4.9:** A fuzzy soft open base for a fuzzy soft topological space is a class of open sets with the property that every fuzzy soft open set is a union of fuzzy soft sets in this class.

i.e. If  $FS_G$  is an arbitrary non-empty fuzzy soft open set and  $FS_{G_e}$  is a fuzzy soft point in  $FS_G$ , then there exists a fuzzy soft set  $FS_B$  in the fuzzy soft open base such that  $FS_{G_e} \quad FS_B \subseteq FS_G$ .

**Definition 4.10:** The fuzzy soft sets in a fuzzy soft open base are referred to as fuzzy soft basic opensets.

**Definition 4.11:** A fuzzy soft open cover of a fuzzy soft topological space whose fuzzy soft sets are all in some given fuzzy soft open base is called a fuzzy soft basic open cover, and if they all lie in some given fuzzy soft open subbase, it is called a fuzzy soft subbasic open cover.

**Note:** We can observe a trivial fact that every fuzzy soft basic open cover in a fuzzy soft compact space contains a finite fuzzy soft subcover.

**Theorem 4.12:** A fuzzy soft topological space is fuzzy soft compact space iff every fuzzy soft basic open cover has a finite fuzzy soft subcover.

**Proof:** Let  $\{FS_{Gi}\}$  be a fuzzy soft open cover and  $\{FS_{Bj}\}$  be a fuzzy soft open base.

Each  $FS_{Gi}$  is the union of certain  $FS_{Bj}$ 's and the totality of all such  $FS_{Bj}$ 's is clearly a fuzzy soft basic open cover.

By our hypothesis, this class of  $FS_{Bj}$ 's has a finite fuzzy subcover.

For each set in this finite fuzzy soft subcover we can select a  $FS_{Gi}$  which contains it by definition 4.9.

The class of  $FS_{Gi}$ 's which arises in this way is evidently a finite fuzzy soft subcover of the original fuzzy soft open cover.

**Definition 4.13:** Let  $(FS_A, T)$  be a fuzzy soft topological space. A class of fuzzy soft closed subsets of  $(FS_A, T)$  is called a fuzzy soft closed base if the class of all complements of its fuzzy soft sets is a fuzzy soft open base and a fuzzy soft closed subbase if the class of all complements is a fuzzy soft open subbase.

**Definition 4.14:** The class of all finite intersections of fuzzy soft sets in a fuzzy soft open subbase is a fuzzy soft open base, it follows that the class of all finite unions of fuzzy soft sets in a fuzzy soft closed subbase is a fuzzy soft closed base. This is called the fuzzy soft closed base generated by the fuzzy soft closed subbase.

**Theorem 4.14:** A fuzzy soft topological space is fuzzy soft compact if every fuzzy soft subbasic open base has a finite fuzzy soft subcover or equivalently if every class of fuzzy soft subbasic closed sets with the finite intersection property has non-empty intersection.

**Proof:** Proof is an easy consequence of theorem 4.6 and theorem 4.8

## V. CONCLUSION

This paper investigates properties of fuzzy soft boundary point and compactness of fuzzy soft topological spaces. Several properties of fuzzy soft open base and fuzzy soft open subbase are discussed. Other concepts can be studied further.

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