An Alternative Elementary Proof for Fermat's Last Theorem

P. N. Seetharaman



Abstract: Fermat's Last Theorem states that the equation x^n $+y^n = z^n$ has no solution forx, y and z as positive integers, where n is any positive integer > 2. Taking the proofs of Fermat and Euler for the exponents n = 4 and n = 3, it would suffice to prove the theorem for the exponent n = p, where p is any prime > 3.We hypothesize that r, s and t are positive integers satisfying the equation $r^p + s^p = t^p$ and establish a contradiction in this proof. We include another Auxiliary equation $x^3 + y^3 = z^3$ and connect these two equations by using transformation equations. On solving the transformation equation we get rst = 0, thus proving that only a trivial solution exists in the main equation $r^p + s^p = t^p$.

Keywords: Transformation Equations to two Fermat's Equations. Mathematics Subject Classification 2010: 11A-XX.

I. INTRODUCTION

Around 1637. Pierre de Fermat French а Mathematician, wrote in the margin of his book, claiming that he has found a marvelous proof for the equation $x^n + y^n$ $=z^n$, but the margin was too narrow to contain it. His proof is available only for the equation $x^4 + y^4 = z^4$, which he had proved using "infinite descent" method. Later on Euler proved the theorem in the equation $x^3 + y^3 = z^3$ [1].

Many mathematicians like Dirichlet, Legendre, Gabril Lame proved the theorem for the exponents n = 5 and n = 7. Around 1820, Sophie Germain gave a remarkable proof for $x^{\ell} + y^{\ell} = z^{\ell}$ where ℓ and $(2\ell + 1)$ are both odd primes and ℓ does not divide xyz [2]. Ernst Kummer made the first substantial step in proving Fermt's Last theorem for Regular Primes [3]. Many mathematicians worked on this theorem by which number theory developed leaps and bounds [4]. Mathematicians found a close relationship between Fermat's Last theorem and Elliptic curve. Finally in 1995 Andrew the Wiles proved theorem completely. Many mathematicians have analysed and explained the theorem in all aspects. In this proof, we are trying for an alternative elementary proof for Fermat's Last theorem.

II. ASSUMPTIONS

1) We presume that all r, s and t arenon-zero positive integers in the equation $r^{p} + s^{p} = t^{p}$ where p is any prime > 3, and establish a contradiction. Gcd (r,s,t) = 1. Any two of r, s and t cannot simultaneously be squares.

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- We are using another auxiliary equation $x^3+y^3=z^3$ 2) (already proved) and we connect the above two equations by means of transformation equations using the parameters called a, b, c, d, e and f.
- 3) Since we are proving the theorem only in the equation $r^{p}+s^{p} =t^{p}$, we have the choice of assigning numerical values for the equation $x^3+y^3 = z^3$. In this proof we give the values x = 29; y = 71; $z^3 = 29^3 + 71^3 = 10^2 \times 3823$ for convenience. gcd(r, s, t) = 1, any two of r, s and t cannot simultaneously be squares.
- We have used F, E and Rin the transformation 4) equations in which we define E and R as distinct odd primes each coprime to each of x, y, z^3 , r. s and t. and F $= 3823 \times rs$
- 5) We may have r, s and t as coprimes to each of 29, 71 and 3823; otherwise we have the choice of assigning alternative values such as x = 11; y = 53; $z^3 = 11^3 + 53^3$ $= 8^2 \times 2347$ such that r, s & t will be coprime to 11, 53 and 2347.

Proof. By trials, we have created the following equations $\left(a\sqrt{t^{p}}+b\sqrt{F^{1/3}}\right)^{2}+\left(c\sqrt{E^{5/3}}+d\sqrt{3823}\right)^{2}=\left(e\sqrt{71}+f\sqrt{R^{1/3}}\right)^{2}$

and

$$\left(a\sqrt{F^{5/3}} - b\sqrt{s^p} \right)^2 + \left(c\sqrt{29} - d\sqrt{E^{1/3}} \right)^2 = \\ \left(e\sqrt{R^{5/3}} - f\sqrt{r^p} \right)^2 \dots (1)$$

to be the transformation equations of $x^3 + y^3 = z^3$ and r^p + $s^p = t^p$ respectively through the parameters called a, b, c, d, e and f. Here we have assigned numerical values for x = 29; y = 71; $z^3 = 29^3 + 71^3 = 10^2 \times 3823.E$ and R are distinct odd primes and $F = (3823rs)^3$. We may have r, s and t as coprimes to 29, 71 and 3823. Otherwise we have the choice of assigning suitable alternative numerical values for x, yand z^3 such that x = 11; y = 53; $z^3 = 11^3 + 53^3 = 8^2 \times 2347$ and so on such that r, s and twill be coprimes to the new odd primes 11, 53 and 2347.

From equation(1), we get

$$a\sqrt{t^{p}} + b\sqrt{F^{1/3}} = \sqrt{x^{3}} \dots (2)$$

$$a\sqrt{F^{5/3}} - b\sqrt{s^{p}} = \sqrt{r^{p}} \dots (3)$$

$$c\sqrt{E^{5/3}} + d\sqrt{3823} = \sqrt{y^{3}} \dots (4)$$

$$c\sqrt{29} - d\sqrt{E^{1/3}} = \sqrt{s^{p}} \dots (5)$$

$$e\sqrt{71} + f\sqrt{R^{1/3}} = \sqrt{z^{3}} \dots (6)$$
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$$e\sqrt{R^{5/3}} - f\sqrt{r^p} = \sqrt{t^p} \quad \dots \quad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$\begin{aligned} a &= \left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) / \left(F + \sqrt{s^{p}t^{p}}\right) \\ b &= \left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right) / \left(F + \sqrt{s^{p}t^{p}}\right) \\ c &= \left(\sqrt{E^{1/3}y^{3}} + \sqrt{3823s^{p}}\right) / \left(E + \sqrt{29 \times 3823}\right) \\ d &= \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right) / \left(E + \sqrt{29 \times 3823}\right) \\ e &= \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right) / \left(R + \sqrt{71r^{p}}\right) \\ and f &= \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right) / \left(R + \sqrt{71r^{p}}\right) \\ From (2) \& (7), we get \\ \sqrt{t^{p}} \times \sqrt{t^{p}} &= \left(\sqrt{x^{3}} - b\sqrt{F^{1/3}}\right) \left(e\sqrt{R^{5/3}} - f\sqrt{r^{p}}\right) / (a) \\ i.e., \quad t^{p} &= \left\{(e)\sqrt{R^{5/3}x^{3}} - (f)\sqrt{r^{p}x^{3}} - (be)\sqrt{F^{1/3}R^{5/3}} + (bf)\sqrt{F^{1/3}r^{p}}\right\} / (a) \end{aligned}$$

From (3) & (5), we have

$$\sqrt{r^{p}} \times \sqrt{r^{p}} = \left(a\sqrt{F^{5/3}} - b\sqrt{s^{p}}\right) \left(e\sqrt{R^{5/3}} - \sqrt{t^{p}}\right) / (f)$$
i.e., $r^{p} = \left\{(ae)\sqrt{F^{5/3}R^{5/3}} - (a)\sqrt{F^{5/3}t^{p}} - (be)\sqrt{R^{5/3}s^{p}} + (b)\sqrt{s^{p}t^{p}}\right\} / (f)$

From (3) & (5), we get

$$\sqrt{s^{p}} \times \sqrt{s^{p}} = \left(a\sqrt{F^{5/3}} - \sqrt{r^{p}}\right) \left(c\sqrt{29} - d\sqrt{E^{1/3}}\right) / (b)$$
i.e., $s^{p} = \left\{(ac)\sqrt{29F^{5/3}} - (ad)\sqrt{F^{5/3}E^{1/3}} - (c)\sqrt{29r^{p}} + (d)\sqrt{E^{1/3}r^{p}}\right\} / (b)$

Substituting the equivalent values of t^p , r^p and s^p in the Fermat's equation $r^p + s^p = t^p$ after multiplying both sides by $\{abf\}$, we get

$$\{bf\} \{(e)\sqrt{R^{5/3}x^3} - (f)\sqrt{x^3r^p} - (be)\sqrt{F^{1/3}R^{5/3}} + (bf)\sqrt{F^{1/3}r^p} \}$$

= $(ab)\{(ae)\sqrt{F^{5/3}R^{5/3}} - (a)\sqrt{F^{5/3}t^p} - (be)\sqrt{R^{5/3}s^p} + (b)\sqrt{s^pt^p} \}$
+ $(af)\{(ac)\sqrt{29F^{5/3}} - (ad)\sqrt{F^{5/3}E^{1/3}} - (c)\sqrt{29r^p} + (d)\sqrt{E^{1/3}r^p} \} (8)$

Our aim is to compute all rational terms in equation (8) and equate them on both sides. To facilitate this, let us multiply both sides of equation (8) by

$$\left\{ \left(F + \sqrt{s^{p}t^{p}}\right)^{3} \left(E + \sqrt{29 \times 3823}\right) \left(R + \sqrt{71r^{p}}\right)^{2} \right\}$$

for freeing from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f*, and again we multiply both sides by $\left(\sqrt{F^{2/3} \times 3823 \times s}\right)$ for getting some rational terms.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b(ef)\}$ = $\sqrt{R^{5/3}x^3} \left(F^2 + s^p t^p + 2F\sqrt{s^p t^p}\right) \left(E + \sqrt{29 \times 3823}\right)$

$$\left(\sqrt{F^{5/3}x^3} - \sqrt{r^pt^p}\right)\left(\sqrt{z^3r^p} + \sqrt{R^{1/3}t^p}\right)\sqrt{F^{2/3} \times 3823 \times s}\left(\sqrt{R^{5/3}z^3} - \sqrt{71t^p}\right)$$

On multiplying by

$$\left\{\sqrt{R^{5/3}x^3}\left(2F\sqrt{s^pt^p}\right)\sqrt{29\times3823}\left(-\sqrt{r^pt^p}\right)\sqrt{R^{1/3}t^p}\sqrt{F^{2/3}\times3823\times s}\left(-\sqrt{71t^p}\right)\right\}$$

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we get

 $\left\{ \left(2FR \times 3823\right) \sqrt{29x^3} \left(t^{2p} \sqrt{s^{p+1}}\right) \sqrt{71r^p} \sqrt{F^{2/3}} \right\}$

which will be irrational.

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II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{bf^2)\}$

=

$$\left(-\sqrt{x^{3}r^{p}}\right)\left(F^{2}+s^{p}t^{p}+2F\sqrt{s^{p}t^{p}}\right)\left(E+\sqrt{29\times3823}\right)\sqrt{F^{2/3}\times3823\times s} \\ \left(\sqrt{F^{5/3}x^{3}}-\sqrt{r^{p}t^{p}}\right)\left(R^{5/3}z^{3}+71t^{p}-2\sqrt{R^{5/3}\times71z^{3}t^{p}}\right)$$

On multiplying by

$$\left(-\sqrt{x^{3}r^{p}}\right)\left(2F\sqrt{s^{p}t^{p}}\right)\sqrt{29\times3823}\sqrt{F^{2/3}\times3823\times s}\left(-\sqrt{r^{p}t^{p}}\right)\left(71t^{p}\right)$$
$$\left\{\left(2\times71F\times3823\right)\sqrt{29x^{3}}\left(t^{2p}r^{p}\sqrt{s^{p+1}}\right)\sqrt{F^{2/3}}\right\}$$

which is irrational.

we get

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b^2(ef)\}$

$$= \left(-\sqrt{F^{1/3}R^{5/3}}\right)\left(F + \sqrt{s^{p}t^{p}}\right)\left(E + \sqrt{29 \times 3823}\right)\sqrt{F^{2/3} \times 3823 \times s}$$
$$\left(F^{5/3}x^{3} + r^{p}t^{p} - 2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right)\left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

(i) on multiplying by

$$\left(\left(-\sqrt{F^{1/3}R^{5/3}}\right)\sqrt{s^{p}t^{p}}\sqrt{29\times3823}\sqrt{F^{2/3}\times3823\times s}\left(-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\sqrt{R^{1/3}t^{p}}\left(-\sqrt{71t^{p}}\right)\right)$$

we get

$$\left\{-(2FR\times 3823)\sqrt{29x^3}\left(t^{2p}\sqrt{s^{p+1}}\right)\sqrt{F^{2/3}}\sqrt{71r^p}\right\}$$

which will be irrational. Also this term gets cancelled with the term worked out under I term in LHS, above. (ii) also on multiplying by

$$\left\{ \left(-\sqrt{F^{1/3}R^{5/3}} \right) \sqrt{s^{p}t^{p}} \left(E \right) \sqrt{F^{2/3} \times 3823 \times s} \left(r^{p}t^{p} \right) \sqrt{R^{1/3}t^{p}} \left(-\sqrt{71t^{p}} \right) \right\}$$
$$\left\{ (ER) \left(r^{p}t^{2p} \sqrt{s^{p+1}} \right) \sqrt{F \times 3823 \times 71t^{p}} \right\}$$

we get

which will be irrational since F = (3823rs) and $\sqrt{71rst^p}$ will be irrational if *r*, *s*&*t* are coprimes to 71. IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b^2f^2\}$

$$=\sqrt{F^{1/3}r^{p}}\left(F+\sqrt{s^{p}t^{p}}\right)\left(E+\sqrt{29\times3823}\right)\sqrt{F^{2/3}\times3823\times s}$$
$$\left(F^{5/3}x^{3}+r^{p}t^{p}-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\left(R^{5/3}z^{3}+71t^{p}-2\sqrt{R^{5/3}\times71z^{3}t^{p}}\right)$$
$$\times\frac{F^{1/3}r^{p}}{2}\sqrt{s^{p}t^{p}}\sqrt{s^{p}t^{p}}\sqrt{29\times3823}\sqrt{F^{2/3}\times3823\times s}\left(-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\left(71t^{p}\right)$$

(i) on multiplying by

$$\sqrt{F^{1/3}r^{p}}\sqrt{s^{p}t^{p}}\sqrt{29\times3823}\sqrt{F^{2/3}\times3823\times s}\left(-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\left(71t^{p}\right)$$

$$\int_{-(2\times71\times3823F)\sqrt{29x^{3}}\left(r^{p}t^{2p}\sqrt{s^{p+1}}\right)\sqrt{F^{2/3}}$$

we get

$$\left\{-(2\times71\times3823F)\sqrt{29x^{3}}\left(r^{p}t^{2p}\sqrt{s^{p+1}}\right)\sqrt{F^{2/3}}\right\}$$

which is irrational.

This rational term gets cancelled with the rational term worked out under II term in LHS above. (ii) also on multiplying by

$$\left\{\sqrt{F^{1/3}r^p}\left(F\times E\right)\sqrt{F^{2/3}\times 3823\times s}\left(r^pt^p\right)\left(71t^p\right)\right\}$$

we get

$$\left\{ \left(71FEr^{p}t^{2p}\right)\sqrt{F\times3823\times r^{p}s}\right\}$$

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which will be rational since F = (3823rs).

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(a^2b)e)\}$

$$= \left(\sqrt{F^{5/3}R^{5/3}}\right) \left(R + \sqrt{s^{p}t^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{F^{2/3} \times 3823 \times s}$$
$$\left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right) \left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right)$$
lying by

on multiplying by

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$$\left\{\sqrt{F^{5/3}R^{5/3}}(R)\sqrt{29\times3823}\sqrt{F^{2/3}\times3823\times s}\left(2\sqrt{F^{1/3}x^3r^ps^p}\right)\sqrt{R^{1/3}t^p}\left(-\sqrt{r^pt^p}\right)\right\}$$
$$\left\{-\left(2FR^2\times3823\right)\sqrt{29x^3}\left(r^pt^p\sqrt{s^{p+1}}\right)\sqrt{F^{2/3}}\right\}$$

which is irrational.

we get

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2b\}$

$$= \left(-\sqrt{F^{5/3}t^{p}}\right) \left(R^{2} + 71r^{p} + 2R\sqrt{71r^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{F^{2/3} \times 3823 \times s} \left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right) \left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right)$$

On multiplying by

$$\left\{ \left(-\sqrt{F^{5/3}t^p} \right) \left(2R\sqrt{71r^p} \right) \sqrt{29 \times 3823} \sqrt{F^{2/3} \times 3823 \times s} \left(-\sqrt{r^p t^p} \right) \left(2\sqrt{F^{1/3}x^3 r^p s^p} \right) \right\} \\ \left\{ \left(4 \times 3823FRr^p t^p \sqrt{s^{p+1}} \right) \sqrt{71x^3} \sqrt{29x^3} \sqrt{r^p \times 71} \sqrt{F^{2/3}} \right\}$$

we get

$$\left\{ \left(4 \times 3823 F R r^p t^p \sqrt{s^{p+1}} \right) \sqrt{71 x^3} \sqrt{29 x^3} \sqrt{r^p \times 71} \sqrt{F^{2/3}} \right\}$$

which will be irrational.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab^2)e\}$

$$= \left(-\sqrt{R^{5/3}s^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{F^{2/3} \times 3823 \times s}$$
$$\left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) \left(F^{5/3}x^{3} + r^{p}t^{p} - 2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right) \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right)$$
ng by

(i) on multiplyin

$$\left\{ \left(-\sqrt{R^{5/3}s^p} \right) (R) \sqrt{29 \times 3823} \sqrt{F^{2/3} \times 3823 \times s} \sqrt{F^{1/3}r^p} \left(-2\sqrt{F^{5/3}x^3r^pt^p} \right) \sqrt{R^{1/3}t^p} \right\}$$

nal term given by

we get the ration

$$\left\{ \left(2 \times 3823 F R^2\right) \sqrt{29 x^3} \left(r^p t^p \sqrt{s^{p+1}}\right) \sqrt{F^{2/3}} \right\}, \text{ which is irrational.}$$

this term get cancelled with the rational term worked out m the I term in the RHS above. (ii) al soon multiplying by

$$\left(-\sqrt{R^{5/3}s^p}\right)(ER)\sqrt{F^{2/3}\times 3823\times s}\sqrt{F^{1/3}r^p}\left(r^pt^p\right)\sqrt{R^{1/3}t^p}\right\}$$

we get

$$\left\{-\left(ER^{2}\right)\left(r^{p}t^{p}\sqrt{s^{p+1}}\right)\sqrt{3823\times Fr^{p}t^{p}}\right\}$$

which will be irrational, since F = (3823rs) and $\sqrt{st^p}$ will be irrational, with gcd(r,t) = 1 and both s&tcan not simultaneously be squares.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ab^2\}$

$$=\sqrt{s^{p}t^{p}}\left(E+\sqrt{29\times3823}\right)\left(R^{2}+71r^{p}+2R\sqrt{71r^{p}}\right)\sqrt{F^{2/3}\times3823\times s}$$
$$\left(\sqrt{x^{3}s^{p}}+\sqrt{F^{1/3}r^{p}}\right)\left(F^{5/3}x^{3}+r^{p}t^{p}-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)$$

(i) On multiplying by

$$\left\{\sqrt{s^{p}t^{p}}\sqrt{29\times3823}\left(2R\sqrt{71r^{p}}\right)\sqrt{F^{2/3}\times3823\times s}\sqrt{F^{1/3}r^{p}}\left(-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\sqrt{29y^{3}}\right\}$$

we get

$$\left\{-\left(4\times 3823FR\right)\sqrt{29x^3}\left(r^pt^p\sqrt{s^{p+1}}\right)\sqrt{71r^p}\sqrt{F^{2/3}}\right\}$$

which is irrational. Also this term gets cancelled with II term RHS the above. (ii) also on multiplying by

$$\left\{\sqrt{s^{p}t^{p}}\left(E\right)\left(R^{2}+71r^{p}\right)\sqrt{F^{2/3}\times3823\times s}\sqrt{F^{1/3}r^{p}}\left(r^{p}t^{p}\right)\right\}$$

we get

$$\left(E\right)\left(r^{p}t^{p}\sqrt{s^{p+1}}\right)\left(R^{2}+71r^{p}\right)\sqrt{F\times 3823r^{p}t^{p}}\right)$$
, which is irrational

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V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2cf\}$

$$= \sqrt{29F^{5/3}} \left(F + \sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{F^{2/3} \times 3823 \times s} \left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{E^{1/3}y^{3}} + \sqrt{3823 \times s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

on multiplying by

$$\left\{\sqrt{29F^{5/3}}\sqrt{s^{p}t^{p}}\sqrt{71r^{p}}\sqrt{F^{2/3}\times 3823\times s}\left(2\sqrt{F^{1/3}}x^{3}r^{p}s^{p}\right)\sqrt{3823\times s^{p}}\left(-\sqrt{71t^{p}}\right)\right\}$$

we get the irrational term given by

$$\left\{-\left(2\times3823\times71Fr^{p}s^{p}t^{p}\sqrt{s^{p+1}}\right)\sqrt{29x^{3}}\sqrt{F^{2/3}}\right\}$$

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2df\}$

$$= \left(-\sqrt{F^{5/3}E^{1/3}}\right) \left(F + \sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{F^{2/3} \times 3823 \times s} \left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

on multiplying by

$$\left\{ \left(-\sqrt{F^{5/3}E^{1/3}} \right) \sqrt{s^{p}t^{p}} \sqrt{71r^{p}} \sqrt{F^{2/3} \times 3823 \times s} \left(2\sqrt{F^{1/3}x^{3}r^{p}s^{p}} \right) \left(-\sqrt{E^{5/3}s^{p}} \right) \left(-\sqrt{71t^{p}} \right) \right\}$$

we get

$$\left\{-\left(2\times71FEr^{p}s^{p}t^{p}\sqrt{s^{p+1}}\right)\sqrt{3823\times x^{3}}\sqrt{F^{2/3}}\right\}$$

which will be irrational, since x = 29 and F = (3823rs).

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a(cf)\}$

$$= \left(-\sqrt{29r^{p}}\right) \left(F^{2} + s^{p}t^{p} + 2F\sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{F^{2/3} \times 3823 \times s} \left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) \left(\sqrt{E^{1/3}y^{3}} + \sqrt{3823 \times s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

on multiplying by

$$\left(-\sqrt{29r^{p}}\right)\left(2F\sqrt{s^{p}t^{p}}\right)\sqrt{71r^{p}}\sqrt{F^{2/3}\times3823\times s}\sqrt{x^{3}s^{p}}\sqrt{3823\times s^{p}}\left(-\sqrt{71t^{p}}\right)\right\}$$

we get the rational term given by

$$\left\{ \left(2 \times 71 \times 3823 F r^p s^p t^p \sqrt{s^{p+1}}\right) \sqrt{29x^3} \sqrt{F^{2/3}} \right\}$$
, which will be irrational.

This term gets cancelled with the rational terms worked out under V term in RHS above. VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{adf\}$

$$= \sqrt{E^{1/3}r^{p}} \left(F^{2} + s^{p}t^{p} + 2F\sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{F^{2/3} \times 3823 \times s} \\ \left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

On multiplying by

$$\left\{\sqrt{E^{1/3}r^{p}}\left(2F\sqrt{s^{p}t^{p}}\right)\sqrt{71r^{p}}\sqrt{F^{2/3}\times 3823\times s}\sqrt{F^{1/3}r^{p}}\left(-\sqrt{E^{5/3}s^{p}}\right)\left(-\sqrt{71t^{p}}\right)\right\}$$

we get

$$\left\{ \left(2 \times 71 F E r^p s^p t^p \right) \sqrt{3823 \times F r^p s} \right\}$$

Which will be rational, since F = (3823rs).

Sum of all rational part in LHS of equation (8)

$$= \left\{ \left(71FEr^{p}t^{2p}\right)\sqrt{F^{1/3} \times 3823 \times r^{p}s} \right\} \text{ (vide IV terms)}$$

Sum of all rational part in RHS of equation (8)

$$= \left\{ \left(2 \times 71 F E r^{p} s^{p} t^{p} \right) \sqrt{3823 \times F^{1/3} r^{p} s} \right\}$$
(vide VIII term)

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 $(71FE)(r^{p}t^{p})\sqrt{3823\times Fr^{p}s}(t^{p}-2s^{p})=0$

 $(71E)(t^p-2s^p)$

Equating the rational terms on both sides of equation (8), We get

Dividing both sides by

we get

$$F(r^{p}t^{p})\sqrt{3823 \times Fr^{p}s} = 0$$

That is, $(3823 \times rs)(r^{p}t^{p})(3823 \times s)\sqrt{r^{p+1}} = 0$ (:: $F = 3823rs$)

That is, either r = 0; or s = 0; or t = 0.

This contradicts our hypothesis that all r, s and t are nonzero integers in the equation $r^p + s^p = t^p$, where p is any primes > 3, thus proving that only a trivial solution exists in the equation.

III. CONCLUSION

Equation (8) was derived from the two transformation equations by substituting the equivalent values of r^p , $s^p \& t^p$, in the Fermat's equation $r^p + s^p = t^p$. The only main hypothesis that we make in the prr of, namely r, s and t are non-zero integers has been shattered by the result rst = 0, that we proving the theorem.

DECLARATION STATEMENT

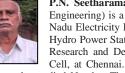
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