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Vibration Normal Modes of a Jib Crane Modeled as an Euler–Bernoulli boom using FEM

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Abstract: ***In this paper, it was developed a method for determining the Vibration Normal Modes of a Jib Crane. A Finite Element Method modeling the Jib Crane as an Euler-Bernoulli boom has been used. We made the approximation of dividing the Boom into a limited number of elements, characterizing the weight distribution on the boom itself. This allowed us to obtain an analytical solution to the problem. The Jib Mass and Stiffness Matrices were calculated. Finally, the first natural frequencies are obtained as well as the first corresponding eigenvectors. From these results, we can derive the behavior of the structural dynamics of the Crane. This is particularly important for large tower cranes that show high structural dynamics, since this approach allows to reduce the vibrations of the crane structure. The advantage of this method is given by the fact that the set of eigen-frequencies can be recalculated using a supervisor Pc. This Pc sends the data of the same eigen-frequencies in real-time to the PLC that controls the crane according to the variable position of the trolley and payload on the Jib.***

Keywords**:** ***Vibrations Normal Modes, Jib Crane, FEM, Euler-Bernoulli Approximation.***

I. INTRODUCTION

A study of the vibration normal modes relative to the structure of the Jib in a Tower crane is presented in this work. Being the Tower cranes structure not rigid, a resulting dynamics vibrations of the crane structure are generated also without the presence of the payload. This effect can cause dangerous instability and serious damages to the crane system. That behavior makes difficult the control of the crane, above all precise positioning and manual control of the crane movement.

Recently, numerous works have been carried out relative to the sway control of a slewing crane with flexible cable for the payload. We can cite [1]-[2]-[3]-[4]-[5], as well as the works of the author of the present paper [6] and [7].

Each of these works corresponds to different control methods with either closed or open loop, as, for example, Adaptive Output Feedback Control, Observer Design for Non-Linear Systems. Also some Patents have been developed regarding the control of the slewing of a tower crane. We can cite [8]. Some of these Patents can be associated with important companies in the Crane sector. Nevertheless, to date, a limited number of papers have considered the contribution of tower crane vibrations to the stability of the slewing movement. The control of rotating Euler-Bernoulli beams has been an active field of research in the last years.

In a recent paper [9], Liu et al. describe and simulate dynamic models to understand tower cranes dynamic characteristics and vibration features. The tower crane is modeled by the finite element method.

Ghazwani et al. [10] defined a Failure Analyses of Tower Crane using FEM. Nevertheless, they studied tower crane's stability during cyclones, without consider the slewing of the crane during the work phase. Also the author of this paper developed a work [11] on the vibration of a Tower crane during the slewing movement. Nevertheless, although an anti-sway solution was developed in this work, the limit was that its applicability might be limited for cranes with large jibs. This effectively also applies to the previous cited works.

Instead, in their paper Rauscher et al. [12] defined a modal method for the slewing control based on a distributed-mass model. The crane jib was modelled as an Euler-Bernoulli beam within a rotating frame of reference. A possible limit of this work is given by the high number of the discretization nodes. In fact, this number should be chosen high enough to precisely model the elastic jib dynamics, hence represents a high order ODE. Therefore, for stabilization they had to perform a complex modal order reduction.

Other relevant works, where Finite Elements Method was applied to Slewing crane, are the cited [13]-[14]-[15]-[16]-[17]-[18].

In this paper, we present an application of vibration analysis of a Jib. As a lot of works on the FEM in analyzing the cranes, we will derive the theoretical background for the governing equations of motion for the jib crane vibration.

The jib crane was treated as a beam conforms to the Euler-Bernoulli beams in order to analyze their vibration properties. The vibration analysis of the jib was investigated using the method of dividing the beam into finite elements, characterized by relevant nodes (counter-jib, trolley, vertical crane). The amplitude of vibration, the first natural frequencies as well as the first mode shapes were calculate using this method. The effect of the different loads (counter-jib, payload, trolley) carried by the jib crane was taking into consideration by generating a localized force on the individual segments on the jib. The resulting theoretical general equations were used to evaluate the vibration parameters of the jib crane using numerical value. This paper is organized as follows. In Section II the dynamical model of the crane is described. The Jib is modeled as an Euler-Bernoulli beam.

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The structure of the Jib is divided into a number of elements of finite size. A set of functions (shape functions) are constructed. The Jib Mass and Stiffness Matrices are calculated. Finally, the first natural frequencies are obtained as well as the first corresponding eigen-vectors. In Section III, an implementation of the method and the most relevant results of the model simulation are presented. Here, we highlight the fact that the set of eigen-frequencies can be recalculated using a supervisor PC, which sends the data of the same eigen-frequencies in real-time to the PLC that controls the crane. At the end, in Section IV concluding remarks are defined.

II. EULER-BERNOULLI BEAM USING FEM

The crane system can be schematizzato as a vertical column, the Tower, a flexible Jib, a trolley system, a hoisting line, and a payload. φ and l represent the slew angle and the length of the hoisting line respectively. The slew angle is the slewing angle of the crane Jib around the Tower or slewing pedestal controlled by the operator's slew command. Sway angles are excited as the system operates, namely the tangential sway φ_t, and the radial sway φ_r. In this study, the payload is regarded as a point mass and the payload exhibits the behavior of a pendulum. A geometric description of the Tower Crane system is given in Fig.1.

As regards the analysis of the elastic deformations of the Jib (referring to papers [19] and [20]) we will make the hypothesis of approximating the Jib to a thin beam, with a load concentrated in well-defined nodes. We will apply the Euler-Bernoulli theory, by ignoring the effects of shear deformation and rotary inertia.

Real systems can be represented as continuous systems with infinite degrees of freedom (d.o.f.). Using continuum theory means dealing with complex partial differential equations. As a consequence, generally, continuous systems with infinite degrees of freedom are discretized into a discrete model with N degrees of freedom which adequately approximates their behavior. There are various discretization techniques, some of which are: the Boundary Element Method, the Multi Body method, the Ritz-Rayleigh method. the Finite Element Method.

For lumped parameter systems with N d.o.f. it is possible to obtain natural frequencies and mode shapes in exact form by equating the determinant of the characteristic equation to zero and solving the system of coupled equations thus obtained. For large values of N, the solution of the system can be long; in these cases it is possible to use the modal method to obtain a system of the same order of magnitude.

The Ritz-Rayleigh (R.R.) method it is essentially a discretization technique for deriving approximate solutions of the system's equation of motion when the displacement v(x,t) is obtained as a linear combination of prescribed functions multiplied by the unknown functions. The latter are obtained by solving an eigenvalue problem. However, for complex systems it is not easy to define a priori a possible shape function over the entire domain. In fact, in R.R.'s method the shape function must be defined over the entire domain of the structure. The finite element method allows us to overcome this problem, in fact the shape functions are defined in small subdomains of the complete system, called finite elements. These functions are low order polynomials and are the same for every finite element. The methodology followed in this work, taken from the finite element method, is the following:

A) The structure of the Jib is divided into a number of elements of finite size. The elements are joined to each other by knots. In our system, each node corresponds to a relevant point of the thin flexible beam with which the Jib is schematized: either it is the fixing point to the ground (knot 0, via the vertical Tower) or that of the final end of the Jib (knot 2), or those where the concentrated masses are defined (knots 1 and 3). In Fig.2 the deformation of the Boom, due to the vibrations in the xy plane, is represented with a dotted line.

Therefore, with reference to Fig.2, we set:

$$L_1 \equiv x_{Tr}; L_2 \equiv L - l_{cj} - x_{Tr}; L_3 \equiv l_{cj} \quad (1)$$

$$\left.\begin{aligned} M_1 &\equiv m_{Tr} + m_L + \left(\mu \cdot x_{Tr}\right) \\ M_2 &\equiv \mu \cdot \left(L - l_{cj} - x_{Tr}\right) \\ M_3 &\equiv m_{cj} + \left(\mu \cdot l_{cj}\right) \end{aligned}\right\} \quad (2)$$

where M_1, M_2, M_3,are the mass corresponding to the positions 1, 2, 3 including the distributed masses respectively on the elements 0-3, 0-1, 1-2, being μ the mass per length unit on the Jib. l_{cj} is the length of the counter-jib, x_{tr} is the trolley position on the x axes on the Jib, L is the total length of the Jib.

B) A given number of d.o.f. is associated with each node. To study the bending vibrations of the beam, each node i will be associated with a displacement $v(x,t)$ along the perpendicular y axis and a rotation $\varphi_z(x,t)$ around the z axis.

C) A set of functions (shape functions) are constructed such that each has a unit value in one degree of freedom and zero in all others. We will represent one-dimensional vectors with the "_" symbol underneath, for example, and two-dimensional matrices with the double subscript, for example. We'll have:

$$v(x,t) = \underline{p}(x) \cdot \underline{\alpha}(t) \equiv \underline{p}(x) \cdot \underline{\underline{A}}^{-1} \cdot \underline{v}(t) \equiv \underline{N}(x) \cdot \underline{v}(t) \quad (3)$$

being:

$$\underline{N}(x) \equiv \underline{p}(x) \cdot \underline{\underline{A}}^{-1} \quad (4)$$

where $\underline{N}(x)$ are the shape functions, $\underline{\underline{A}}$ is the Coefficient matrix. We will have, in fact:

$$\underline{p}(x) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \quad (5)$$

$$\underline{\alpha}^T(t) = \begin{bmatrix} \alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix} \quad (6)$$

being $\underline{\alpha}(t)$ the vector of the time coefficients. It turns out like this:

$$\varphi_z(x,t) = \frac{d}{dx} v\left(x,t\right) \equiv \begin{bmatrix} 0 & 1 & 2x & 3x^2 \end{bmatrix} \cdot \underline{\alpha}(t) \quad (7)$$

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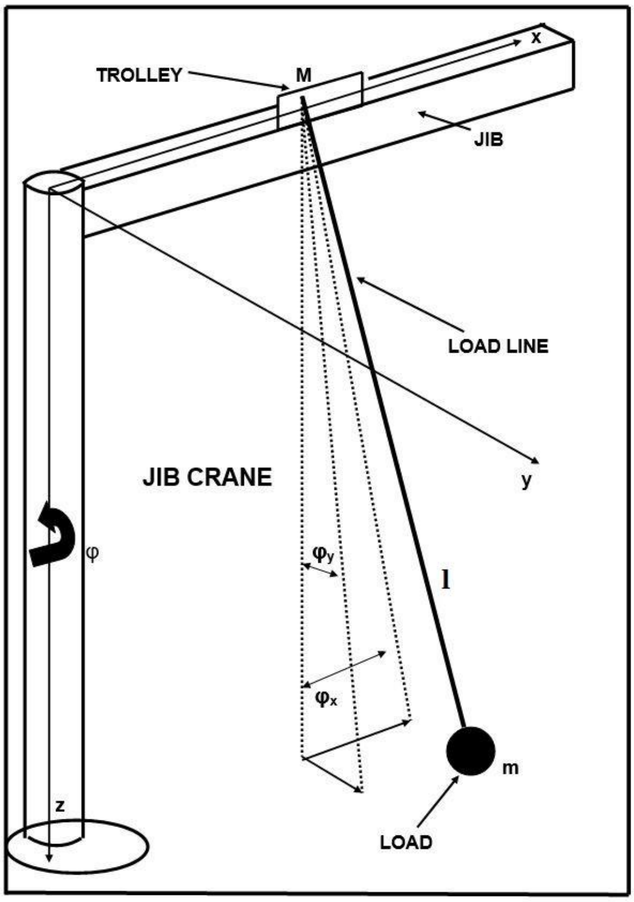

Fig. 1. Geometric Description of the Tower Crane System

A. Segment 03

Let's now define the individual segments into which the Jib has been divided. With reference to Fig.2, regarding the Segment 03, we set:

$$\underline{v}_{03}^{T} = \left[v_0 \quad \varphi_{0z} \quad v_3 \quad \varphi_{3z} \right] \tag{8}$$

$$v(0) = \alpha_0; \qquad \varphi_z(0) = \alpha_1 \tag{9}$$

$$v(-L_3) = \alpha_0 - \alpha_1 L_3 + \alpha_2 L_3^2 - \alpha_3 L_3^3; \tag{10}$$

$$\varphi_z(-L_3) = \alpha_1 - 2\alpha_2 L_3 + 3\alpha_3 L_3^2; \tag{11}$$

$$\underline{\underline{A}}_{03} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -L_3 & L_3^2 & -L_3^3 \\ 0 & 1 & -2L_3 & 3L_3^2 \end{pmatrix} \tag{12}$$

$$\det\left(\underline{\underline{A}}_{03}\right) = L_3^4 \tag{13}$$

$$\underline{v}_{03} = \underline{\underline{A}}_{03} \cdot \underline{\alpha}(t) \Rightarrow \underline{\alpha}(t) = \underline{\underline{A}}_{03}^{-1} \cdot \underline{v}_{03} \tag{14}$$

$$A_{11} = \det \begin{pmatrix} 1 & 0 & 0 \\ -L_3 & L_3^2 & -L_3^3 \\ 1 & -2L_3 & 3L_3^2 \end{pmatrix} \equiv 3L_3^4 - 2L_3^4 \equiv L_3^4 \tag{15}$$

$$\underline{\underline{A}}_{03}^{*} \equiv Cof_{03}^{T} = \begin{pmatrix} L_3^4 & 0 & 0 & 0 \\ 0 & L_3^4 & 0 & 0 \\ -3L_3^2 & 2L_3^3 & 3L_3^2 & L_3^3 \\ -2L_3 & L_3^2 & 2L_3 & L_3^2 \end{pmatrix} \tag{16}$$

where $\underline{\underline{A}}_{03}^{*}$ is the self-adjoint matrix of the matrix $\underline{\underline{A}}$.

We thus obtain the inverse matrix $\underline{\underline{A}}_{03}^{-1}$

$$\underline{\underline{A}}_{03}^{-1} \equiv \frac{1}{\left(\det \underline{\underline{A}}_{03}\right)} \underline{\underline{A}}_{03}^{*} = \frac{\underline{\underline{A}}_{03}^{*}}{L_3^4} \tag{17}$$

and the resulting shape vector $\underline{N}_{03}$

$$\underline{N}_{03} = \underline{p}_{03} \cdot \underline{\underline{A}}_{03}^{-1} \equiv \left[1 \quad x \quad x^2 \quad x^3 \right] \cdot \underline{\underline{A}}_{03}^{-1} \tag{18}$$

$$\underline{N}_{03}^{T} = \begin{bmatrix} \left(1 - \dfrac{3x^2}{L_3^2} - \dfrac{2x^3}{L_3^3}\right) \\ \left(x + \dfrac{2x^2}{L_3} + \dfrac{x^3}{L_3^2}\right) \\ \left(\dfrac{3x^2}{L_3^2} + \dfrac{2x^3}{L_3^3}\right) \\ \left(\dfrac{x^2}{L_3} + \dfrac{x^3}{L_3^2}\right) \end{bmatrix} \tag{19}$$

D) The shape functions of an element are substituted into the expression of the kinetic energy T_{ij} and the elastic potential energy to obtain the Mass and Stiffness matrices of each finite element U_{ij} to obtain the Mass $\underline{\underline{M}}_{ij}$ and Stiffness $\underline{\underline{K}}_{ij}$ matrices of each finite element. Thus, from the Kinetic Energy T_{03} of element 03is obtained the expression of the Mass $\underline{\underline{M}}_{03}$:

$$T_{03} = \frac{1}{2} \int_0^{-L_3} \mu \underline{\underline{A}}_{03} \dot{\underline{v}}_{03}^2(x,t) dx$$
$$\underline{\underline{M}}_{03} = \mu \cdot \int_0^{-L_3} \underline{N}_{03}^{T}(x) \cdot \underline{N}_{03}(x) dx \tag{20}$$

being μ the mass per unit length on the beam.

$$\underline{\underline{M}}_{03} = \mu \cdot \begin{pmatrix} \left(-\frac{13}{35}\right) L_3 & \left(\frac{11}{210}\right) L_3^2 & \left(-\frac{9}{70}\right) L_3 & \left(\frac{47}{420}\right) L_3^2 \\ \left(\frac{11}{210}\right) L_3^2 & \left(-\frac{1}{105}\right) L_3^3 & \left(\frac{13}{420}\right) L_3^2 & \left(\frac{1}{140}\right) L_3^3 \\ \left(-\frac{9}{70}\right) L_3 & \left(\frac{13}{420}\right) L_3^2 & \left(-\frac{13}{35}\right) L_3 & \left(-\frac{11}{210}\right) L_3^2 \\ \left(\frac{47}{420}\right) L_3^2 & \left(\frac{1}{140}\right) L_3^3 & \left(-\frac{11}{210}\right) L_3^2 & \left(-\frac{1}{105}\right) L_3^3 \end{pmatrix} \tag{21}$$

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$$\underline{\underline{M}}_3 = \left(m_{cj}\cdot \underline{\underline{I}}\right) + \underline{\underline{M}}_{03} \tag{22}$$

being m_{cj} the mass of the counter-jib and $\underline{\underline{I}}$ the Identity matrix. It is also obtained, from Elastic Potential Energy U_{03} of the element 03, the expression of the Stiffness matrix $\underline{\underline{K}}_{03}$:

$$\begin{gathered} U_{03} = \frac{1}{2}\int_0^{-L_3} EI_z\left(\frac{\partial^2 \underline{v}_{03}(x,t)}{\partial x^2}\right)^2 dx \\ \underline{\underline{K}}_{03} = EI_z\cdot\int_0^{-L_3}\left\{\frac{d^2\underline{N}_{03}(x)}{dx^2}\right\}^T\left\{\frac{d^2\underline{N}_{03}(x)}{dx^2}\right\}dx \end{gathered} \tag{23}$$

being E the elastic modulus and I_z the second moment of inertia on the boom.

$$\left\{\frac{d^2\underline{N}_{03}(x)}{dx^2}\right\}^T = \begin{bmatrix} -\left(\frac{6}{L_3^2}+\frac{12x}{L_3^3}\right) \\ \left(\frac{4}{L_3}+\frac{6x}{L_3^2}\right) \\ \left(\frac{6}{L_3^2}+\frac{12x}{L_3^3}\right) \\ \left(\frac{2}{L_3}+\frac{6x}{L_3^2}\right) \end{bmatrix} \tag{24}$$

$$\underline{\underline{K}}_{03} = EI_z\cdot\begin{pmatrix} \left(-\frac{12}{L_3^3}\right) & \left(\frac{78}{L_3^2}\right) & \left(\frac{12}{L_3^3}\right) & \left(-\frac{6}{L_3^2}\right) \\ \left(\frac{78}{L_3^2}\right) & 0 & \left(-\frac{78}{L_3^2}\right) & \left(-\frac{2}{L_3}\right) \\ \left(\frac{12}{L_3^3}\right) & \left(-\frac{78}{L_3^2}\right) & \left(-\frac{12}{L_3^3}\right) & \left(\frac{6}{L_3^2}\right) \\ \left(-\frac{6}{L_3^2}\right) & \left(-\frac{2}{L_3}\right) & \left(\frac{6}{L_3^2}\right) & \left(-\frac{4}{L_3}\right) \end{pmatrix} \tag{25}$$

B. Segment 01

As regards the Segment 01, in a similar way, we will have:

$$\underline{v}_{01}^T = \begin{bmatrix} v_0 & \varphi_{0z} & v_1 & \varphi_{1z}\end{bmatrix} \tag{26}$$

$$v(0) = \alpha_0; \qquad \varphi_z(0) = \alpha_1 \tag{27}$$

$$v(L_1) = \alpha_0 + \alpha_1 L_1 + \alpha_2 L_1^2 + \alpha_3 L_1^3; \tag{28}$$

$$\varphi_z(L_1) = \alpha_1 + 2\alpha_2 L_1 + 3\alpha_3 L_1^2; \tag{29}$$

$$\underline{\underline{A}}_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L_1 & L_1^2 & L_1^3 \\ 0 & 1 & 2L_1 & 3L_1^2 \end{pmatrix} \tag{30}$$

$$\det\left(\underline{\underline{A}}_{01}\right) = L_1^4 \tag{31}$$

$$\underline{v}_{01} = \underline{\underline{A}}_{01}\cdot\underline{\alpha}(t) \Rightarrow \underline{\alpha}(t) = \underline{\underline{A}}_{01}^{-1}\cdot\underline{v}_{01} \tag{32}$$

$$A_{14} = \det\begin{pmatrix} 0 & 1 & 0 \\ 1 & L_1 & L_1^2 \\ 0 & 1 & 2L_1 \end{pmatrix} \equiv -2L_1 \tag{33}$$

$$\underline{\underline{A}}_{01}^* \equiv Cof_{01}^T = \begin{pmatrix} L_1^4 & 0 & 0 & 0 \\ 0 & L_1^4 & 0 & 0 \\ 3L_1^2 & -2L_1^3 & 3L_1^2 & -L_1^3 \\ 2L_1 & L_1^2 & -2L_1 & L_1^2 \end{pmatrix} \tag{34}$$

$$\underline{\underline{A}}_{01}^{-1} \equiv \frac{1}{\left(\det \underline{\underline{A}}_{01}\right)}\underline{\underline{A}}_{01}^* = \frac{\underline{\underline{A}}_{01}^*}{L_1^4}$$

$$\underline{N}_{01} = \underline{p}_{01}\cdot\underline{\underline{A}}_{01}^{-1} \equiv \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix}\cdot\underline{\underline{A}}_{01}^{-1} \tag{35}$$

$$\underline{N}_{01}^T = \begin{bmatrix} \left(1+\frac{3x^2}{L_1^2}+\frac{2x^3}{L_1^3}\right) \\ \left(x-\frac{2x^2}{L_1}+\frac{x^3}{L_1^2}\right) \\ \left(\frac{3x^2}{L_1^2}-\frac{2x^3}{L_1^3}\right) \\ \left(-\frac{x^2}{L_1}+\frac{x^3}{L_1^2}\right) \end{bmatrix} \tag{36}$$

$$\underline{\underline{M}}_{01} = \mu\cdot\int_0^{L_1}\underline{N}_{01}^T(x)\cdot\underline{N}_{01}(x)dx \tag{37}$$

$$\underline{\underline{M}}_{01} = \mu\cdot\begin{pmatrix} \left(\frac{59}{7}\right)L_1 & \left(\frac{67}{210}\right)L_1^2 & \left(\frac{13}{7}\right)L_1 & \left(-\frac{31}{60}\right)L_1^2 \\ \left(\frac{67}{210}\right)L_1^2 & \left(\frac{1}{105}\right)L_1^3 & \left(-\frac{11}{210}\right)L_1^2 & \left(-\frac{1}{140}\right)L_1^3 \\ \left(\frac{13}{7}\right)L_1 & \left(-\frac{11}{210}\right)L_1^2 & \left(\frac{13}{35}\right)L_1 & \left(-\frac{11}{210}\right)L_1^2 \\ \left(-\frac{47}{420}\right)L_1^2 & \left(-\frac{1}{140}\right)L_1^3 & \left(-\frac{11}{210}\right)L_1^2 & \left(\frac{1}{105}\right)L_1^3 \end{pmatrix} \tag{38}$$

$$\underline{\underline{M}}_1 = \left[\left(m_{Tr}+m_L\right)\cdot\underline{\underline{I}}\right] + \underline{\underline{M}}_{01} \tag{39}$$

being m_{Tr} the Trolley mass and m_L the Load mass.

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$$\left\{\frac{d^2\underline{N}_{01}(x)}{dx^2}\right\}^T = \begin{bmatrix} \left(\frac{6}{L_1^2}+\frac{12x}{L_1^3}\right) \\ \left(-\frac{4}{L_1}+\frac{6x}{L_1^2}\right) \\ \left(\frac{6}{L_1^2}-\frac{12x}{L_1^3}\right) \\ \left(-\frac{2}{L_1}+\frac{6x}{L_1^2}\right) \end{bmatrix} \quad (40)$$

$$\underline{\underline{K}}_{01} = EI_z \cdot \int_0^{L_1} \left\{\frac{d^2\underline{N}_{01}(x)}{dx^2}\right\}^T \left\{\frac{d^2\underline{N}_{01}(x)}{dx^2}\right\} dx \quad (41)$$

$$\underline{\underline{K}}_{01} = EI_z \cdot \begin{pmatrix} \left(\frac{156}{L_1^3}\right) & \left(-\frac{54}{L_1^2}\right) & \left(-\frac{12}{L_1^3}\right) & \left(\frac{18}{L_1^2}\right) \\ \left(-\frac{54}{L_1^2}\right) & 0 & \left(-\frac{6}{L_1^2}\right) & \left(\frac{2}{L_1}\right) \\ \left(-\frac{12}{L_1^3}\right) & \left(-\frac{6}{L_1^2}\right) & \left(\frac{12}{L_1^3}\right) & \left(-\frac{6}{L_1^2}\right) \\ \left(\frac{18}{L_1^2}\right) & \left(\frac{2}{L_1}\right) & \left(-\frac{6}{L_1^2}\right) & \left(\frac{4}{L_1}\right) \end{pmatrix} \quad (42)$$

C. Segment 12

Concerning the Segment 12, we get the same results as Segment 01, simply by replacing the length L_1 with the length L_2. This is except for the boundary conditions that are set subsequently. We will therefore have:

$$\underline{\underline{M}}_2 = \underline{\underline{M}}_{12} \quad (43)$$

D. Matrices M and K for the Overall Beam (Jib)

Let $\underline{v}$ the vector containing all the d.o.f. of the 3-element beam considered:

$$\underline{v}^T = \begin{bmatrix} v_3 & \vartheta_{z3} & v_0 & \vartheta_{z0} & v_1 & \vartheta_{z1} & v_2 & \vartheta_{z2} \end{bmatrix} \quad (44)$$

This vector can be related to the vectors $\underline{v}_{ij}$ relating to the individual finite elements using the transformation matrices $\underline{\underline{a}}_{ij}$. We will therefore have for the 3 segments:

$$\underline{\underline{a}}_{30} = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{pmatrix}; \underline{v}_{30} = \begin{bmatrix} v_3 \\ \vartheta_{z3} \\ v_0 \\ \vartheta_{z0} \end{bmatrix} \quad (45)$$

$$\underline{\underline{a}}_{01} = \begin{pmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{pmatrix}; \underline{v}_{01} = \begin{bmatrix} v_0 \\ \vartheta_{z0} \\ v_1 \\ \vartheta_{z1} \end{bmatrix} \quad (46)$$

$$\underline{\underline{a}}_{12} = \begin{pmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}; \underline{v}_{12} = \begin{bmatrix} v_{01} \\ \vartheta_{z1} \\ v_2 \\ \vartheta_{z2} \end{bmatrix} \quad (47)$$

being:

$$\begin{cases} \underline{\underline{a}}_{30} \cdot \underline{v} = \underline{v}_{30} \\ \underline{\underline{a}}_{01} \cdot \underline{v} = \underline{v}_{01} \\ \underline{\underline{a}}_{12} \cdot \underline{v} = \underline{v}_{12} \end{cases} \quad (48)$$

Now we consider the matrix $\underline{\underline{M}}_{03}$ for the segment 30 which we describe (for synthetic reasons) using the symbols a_{ij} for its elements:

$$\underline{\underline{M}}_{03} = \mu \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \quad (49)$$

The matrix $\underline{\underline{M}}_{03}$ will be associated with the total matrix $\underline{\underline{M}}_{TOT(0)}$ of the distributed masses, using the following transformation:

$$\underline{\underline{a}}_{30}^T \underline{\underline{M}}_{03} \underline{\underline{a}}_{30} = \mu \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0&0&0&0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0&0&0&0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0&0&0&0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad (50)$$

Similarly, we will consider the matrix $\underline{\underline{M}}_{01}$ for the segment 01 using the symbols b_{ij} for its elements; we will have:

$$\underline{\underline{M}}_{01} = \mu \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \quad (51)$$

The matrix $\underline{\underline{M}}_{01}$ will be associated with the total matrix $\underline{\underline{M}}_{TOT(0)}$ of the distributed masses using the following transformation:

$$\underline{\underline{a}}_{01}^{T}\underline{\underline{M}}_{01}\underline{\underline{a}}_{01} = \mu\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{11} & b_{12} & b_{13} & b_{14} & 0 & 0 \\ 0 & 0 & b_{21} & b_{22} & b_{23} & b_{24} & 0 & 0 \\ 0 & 0 & b_{31} & b_{32} & b_{33} & b_{34} & 0 & 0 \\ 0 & 0 & b_{41} & b_{42} & b_{43} & b_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (52)$$

Similarly, we will consider the matrix $\underline{\underline{M}}_{12}$ for the segment 12 using the symbols c_{ij} for its elements; we will have:

$$\underline{\underline{M}}_{12} = \mu\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix} \quad (53)$$

The matrix $\underline{\underline{M}}_{12}$ will be associated with the total $\underline{\underline{M}}_{TOT(0)}$ of the distributed masses using the following transformation:

$$\underline{\underline{a}}_{12}^{T}\underline{\underline{M}}_{12}\underline{\underline{a}}_{12} = \mu\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{11} & c_{12} & c_{13} & c_{14} \\ 0 & 0 & 0 & 0 & c_{21} & c_{22} & c_{23} & c_{24} \\ 0 & 0 & 0 & 0 & c_{31} & c_{32} & c_{33} & c_{34} \\ 0 & 0 & 0 & 0 & c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix} \quad (54)$$

$$\underline{\underline{M}}_{TOT(0)} = \mu\begin{pmatrix} \left(-\frac{13}{35}\right)L_3 & \left(\frac{11}{210}\right)L_3^2 & \left(-\frac{9}{70}\right)L_3 & \left(\frac{47}{420}\right)L_3^2 & 0 & 0 & 0 & 0 \\ \left(\frac{11}{210}\right)L_3^2 & \left(-\frac{1}{105}\right)L_3^3 & \left(\frac{13}{420}\right)L_3^2 & \left(\frac{1}{140}\right)L_3^3 & 0 & 0 & 0 & 0 \\ \left(-\frac{9}{70}\right)L_3 & \left(\frac{13}{420}\right)L_3^2 & \left[\left(-\frac{13}{35}\right)L_3+\left(\frac{59}{7}\right)L_1\right] & \left[\left(-\frac{11}{210}\right)L_3^2+\left(\frac{67}{210}\right)L_1^2\right] & \left(\frac{13}{7}\right)L_1 & \left(-\frac{31}{60}\right)L_1^2 & 0 & 0 \\ \left(\frac{47}{420}\right)L_3^2 & \left(\frac{1}{140}\right)L_3^3 & \left[\left(-\frac{11}{210}\right)L_3^2+\left(\frac{67}{210}\right)L_1^2\right] & \left[\left(-\frac{1}{105}\right)L_3^3+\left(\frac{1}{105}\right)L_1^3\right] & \left(-\frac{11}{210}\right)L_1^2 & \left(-\frac{1}{140}\right)L_1^3 & 0 & 0 \\ 0 & 0 & \left(\frac{13}{7}\right)L_1 & \left(-\frac{11}{210}\right)L_1^2 & \left[\left(\frac{13}{35}\right)L_1+\left(\frac{59}{7}\right)L_2\right] & \left[\left(-\frac{11}{210}\right)L_1^2+\left(\frac{67}{210}\right)L_2^2\right] & \left(\frac{13}{7}\right)L_2 & \left(-\frac{31}{60}\right)L_2^2 \\ 0 & 0 & \left(-\frac{31}{60}\right)L_1^2 & \left(-\frac{1}{140}\right)L_1^3 & \left[\left(-\frac{11}{210}\right)L_1^2+\left(\frac{67}{210}\right)L_2^2\right] & \left[\left(\frac{1}{105}\right)L_1^3+\left(\frac{1}{105}\right)L_2^3\right] & \left(-\frac{11}{210}\right)L_2^2 & \left(-\frac{1}{140}\right)L_2^3 \\ 0 & 0 & 0 & 0 & \left(\frac{13}{7}\right)L_2 & \left(-\frac{11}{210}\right)L_2^2 & \left(\frac{13}{35}\right)L_2 & \left(-\frac{11}{210}\right)L_2^2 \\ 0 & 0 & 0 & 0 & \left(-\frac{31}{60}\right)L_2^2 & \left(-\frac{1}{140}\right)L_2^3 & \left(-\frac{11}{210}\right)L_2^2 & \left(\frac{1}{105}\right)L_2^3 \end{pmatrix} \quad (55)$$

The total matrix on the Jib $\underline{\underline{M}}_{TOT(0)}$, relating to the distributed masses, was obtained as:

$$\underline{\underline{M}}_{TOT(0)} = \left(\underline{\underline{a}}_{30}^{T}\underline{\underline{M}}_{03}\underline{\underline{a}}_{30} + \underline{\underline{a}}_{01}^{T}\underline{\underline{M}}_{01}\underline{\underline{a}}_{01} + \underline{\underline{a}}_{12}^{T}\underline{\underline{M}}_{12}\underline{\underline{a}}_{12}\right). \quad (56)$$

From which $\underline{\underline{M}}_{TOT}$ will turn out to be, coming form (22), (39) e (43):

$$\underline{\underline{M}}_{TOT} = \underline{\underline{M}}_{1} + \underline{\underline{M}}_{2} + \underline{\underline{M}}_{3} \quad (57)$$

We will proceed in a completely similar way for the stiffness matrix $\underline{\underline{K}}_{TOT(0)}$, relative to the distributed masses, obtaining:

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$$\underline{\underline{K}}_{TOT(0)} = EI_z \begin{pmatrix} \left(-\frac{12}{L_3^3}\right) & \left(\frac{78}{L_3^2}\right) & \left(\frac{12}{L_3^3}\right) & \left(-\frac{6}{L_3^2}\right) & 0 & 0 & 0 & 0 \\ \left(\frac{78}{L_3^2}\right) & 0 & \left(-\frac{78}{L_3^2}\right) & \left(-\frac{2}{L_3}\right) & 0 & 0 & 0 & 0 \\ \left(\frac{12}{L_3^3}\right) & \left(-\frac{78}{L_3^2}\right) & \left[\left(-\frac{12}{L_3^3}\right)+\left(\frac{156}{L_1^3}\right)\right] & \left[\left(\frac{6}{L_3^2}\right)+\left(-\frac{54}{L_1^2}\right)\right] & \left(-\frac{12}{L_1^3}\right) & \left(\frac{18}{L_1^2}\right) & 0 & 0 \\ \left(-\frac{6}{L_3^2}\right) & \left(-\frac{2}{L_3}\right) & \left[\left(\frac{6}{L_3^2}\right)+\left(-\frac{54}{L_1^2}\right)\right] & \left[\left(-\frac{4}{L_3}\right)+0\right] & \left(-\frac{6}{L_1^2}\right) & \left(\frac{2}{L_1}\right) & 0 & 0 \\ 0 & 0 & \left(-\frac{12}{L_1^3}\right) & \left(-\frac{6}{L_1^2}\right) & \left[\left(\frac{12}{L_1^3}\right)+\left(\frac{156}{L_2^3}\right)\right] & \left[\left(-\frac{6}{L_1^2}\right)+\left(-\frac{54}{L_2^2}\right)\right] & \left(-\frac{12}{L_2^3}\right) & \left(\frac{18}{L_2^2}\right) \\ 0 & 0 & \left(\frac{18}{L_1^2}\right) & \left(\frac{2}{L_1}\right) & \left[\left(-\frac{6}{L_1^2}\right)+\left(-\frac{54}{L_2^2}\right)\right] & \left[\left(\frac{4}{L_1}\right)+0\right] & \left(-\frac{6}{L_2^2}\right) & \left(\frac{2}{L_2}\right) \\ 0 & 0 & 0 & 0 & \left(-\frac{12}{L_2^3}\right) & \left(-\frac{6}{L_2^2}\right) & \left(\frac{12}{L_2^3}\right) & \left(-\frac{6}{L_2^2}\right) \\ 0 & 0 & 0 & 0 & \left(\frac{18}{L_2^2}\right) & \left(\frac{2}{L_2}\right) & \left(-\frac{6}{L_2^2}\right) & \left(\frac{4}{L_2}\right) \end{pmatrix} \quad (58)$$

The total stiffness matrix $\underline{\underline{K}}_{TOT(0)}$ relating to the distributed masses, is obtained as:

$$\underline{\underline{K}}_{TOT} \equiv \underline{\underline{K}}_{TOT(0)} = \left(\underline{\underline{a}}_{30}^T \underline{\underline{K}}_{03} \underline{\underline{a}}_{30} + \underline{\underline{a}}_{01}^T \underline{\underline{K}}_{01} \underline{\underline{a}}_{01} + \underline{\underline{a}}_{12}^T \underline{\underline{K}}_{12} \underline{\underline{a}}_{12}\right) \quad (59)$$

In a next work we will describe how this method of determining the normal modes of vibration for the Jib can be applied to control the vibrations and sway of a Slewing Crane.

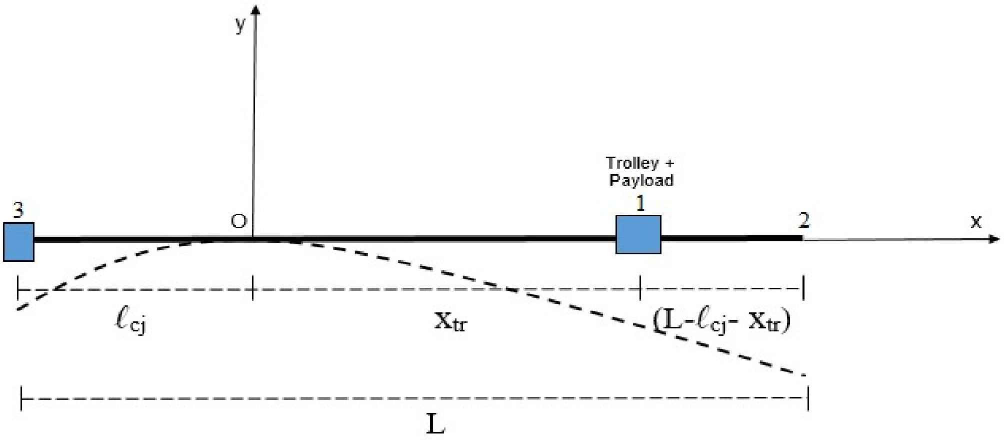

Fig. 2 Finite Elements Distribution on the Jib

III. RESULT AND DISCUSSION

We are now able to calculate the eigenvalues and eigenvectors associated with the dynamic vibration problem. Some examples of the determination of Natural Frequencies and Vibration Measurement are given in articles [21] and [22]. We use GNU Octave version 5.1.0. Octave is a mathematical package capable of solving an eigenvalue and eigenvector problem. In particular for this type of problem, the syntax is:

[eigenvector,eigenval] = eig(Square_Matrix).

If we suppose we have a 2 degrees of freedom system of which we know the stiffness and mass matrix, with Octave we can perform the modal analysis using the following syntax:

$$\left[\underline{\underline{K}}_{TOT} - \omega_i^2 \underline{\underline{M}}_{TOT}\right] \cdot \Phi_i = \underline{0} \quad (60)$$

Clearly $\underline{\underline{K}}_{TOT}$ e $\underline{\underline{M}}_{TOT}$ are known terms, as they are derived a priori from the characteristics of the structural system, while Φ_i e ω_i are unknowns. The previous expression can be rewritten as follows:

$$\underline{\underline{K}}_{TOT} \cdot \Phi_i = \omega_i^2 \underline{\underline{M}}_{TOT} \cdot \Phi_i \ . \quad (61)$$

This represents a generalized eigenvalue and eigenvector problem and can be reduced to the standard form simply by pre-multiplying by the inverse matrix $\underline{\underline{M}}_{TOT}$. We will have:

$$\underline{\underline{M}}_{TOT}^{-1} \cdot \underline{\underline{K}}_{TOT} \cdot \underline{\Phi}_i = \omega_i^2 \cdot \underline{\Phi}_i \qquad (62)$$

It is therefore a matter of finding the eigenvalues and eigenvectors of the overall matrix $\underline{\underline{M}}_{TOT}^{-1} \cdot \underline{\underline{K}}_{TOT}$.

In Tab.1 are described the Exemplary Crane Parameters for a Jib of a large Crane.

In the following figures the eigenvector 1 (Fig.3), eigenvector 2 (Fig.4), eigenvector 3 (Fig.5) and eigenvector 4 (Fig.5) are represented. Finally, the histogram relating to the Eigenfrequencies corresponding to the first six Vibration Norman Modes is defined in Fig.7.

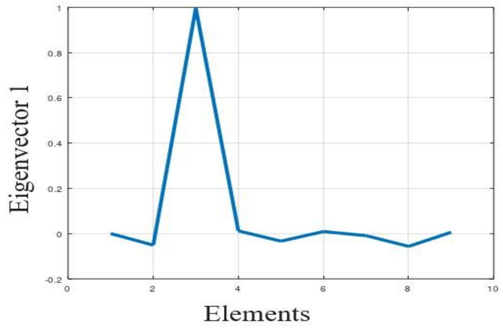

Fig. 3 Eigenvector 1

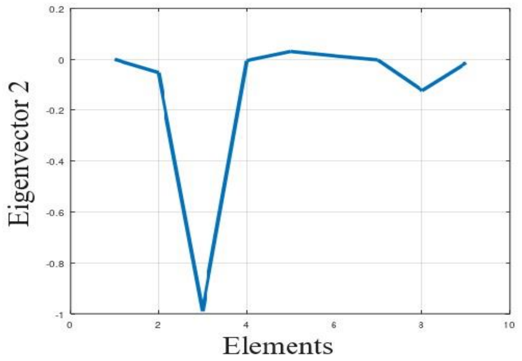

Fig. 4 Eigenvector 2

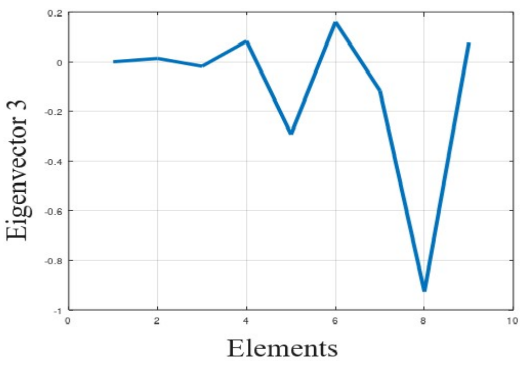

Fig. 5 Eigenvector 3

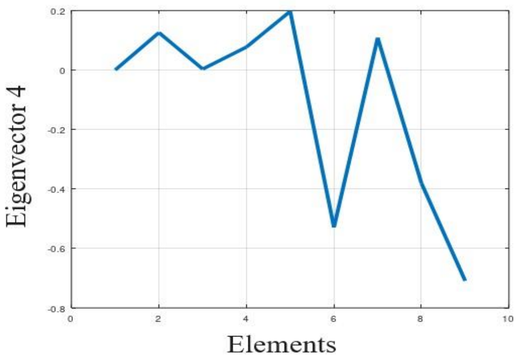

Fig. 6 Eigenvector 4

Once we have obtained the eigen-frequencies we can define what the relevant positive aspect of this new method is. In fact, since the calculation of the eigen-frequencies is carried out in analytical form, they can be recalculated in real-time using a PC-supervisor. This will send, upon request, to the PLC that controls the crane, the data of the same eigen-frequencies depending on the variable position on the Jib of the trolley and the payload. In this way the speed profile, calculated on the PLC to obtain the anti-vibration and anti-sway effect, will always be correlated to the actual position of the trolley (and the payload) on the Jib. This position can obviously be very different depending on the situations. Otherwise, in the case of using a FEM with a very high number of elements (which happens with the methods described by the previous works) this calculation cannot be carried out in real-time, and therefore the possibility of having a description of the real system is compromised.

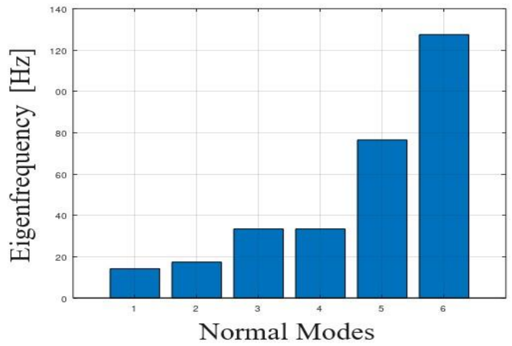

Fig.7 Eigenfrequencies Corresponding to the first 6 Vibration Norman Modes

Table I: Exemplary Crane Parameters

Symbol	Parameter	Value	Unit
μ	Linear density (Mass per length unit)	100	kg/m
E	Elastic Modulus	$210 \cdot 10^9$	Pa
I_z	Second Moment of Inertia	$5 \cdot 10^{-3}$	m^4
L	Length of the Jib	60	m
l_{CJ}	Length of the counter-jib	12	m
X_{Tr}	Variable position of the Trolley on the Jib	30	m
m_{CJ}	Counter-Jib mass	$5 \cdot 10^3$	kg
m_L	Load mass	$2 \cdot 10^3$	kg
m_{Tr}	Trolley mass	$6 \cdot 10^3$	kg

IV. CONCLUSION

This paper explores an effective method of analyzing the vibration of a jib crane by utilizing a Finite Element Method. The Jib Crane was modelled as an Euler-Bernoulli boom with the aim of finding the amplitude, frequency and mode shapes. The Boom was divided into a limited number of elements, characterizing the distribution of weights on the boom itself, in order to obtain an analytical solution to the problem. An approximate mode equation was derived using expansion of the modes shape as the expansion function for the beam. The results show an effective and practical approach to the computation of vibration of Jib structure, particularly important for large tower cranes that present high structural dynamics.

The advantageous novelty of this method is given by the fact that the eigen-frequencies can be calculated in real time, using a supervisor PC, which sends the data of the same eigen-frequencies to the PLC that controls the Jib crane. Therefore, these data are according to the variable position on the Jib of the trolley and payload, so that the PLC determines the most correct anti-vibration speed profiles.

DECLARATION STATEMENT

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Conflicts of Interest	No conflicts of interest to the best of our knowledge.
Ethical Approval and Consent to Participate	No, the article does not require ethical approval and consent to participate with evidence.
Availability of Data and Material	Not relevant.
Authors Contributions	I am only the sole author of the article.

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