International Journal of Basic Sciences and Applied Computing (IJBSAC)
ISSN: 2394-367X, Volume-1 Issue-4, February 2015

Prime Radicals and Completely Prime Radicals in Ternary Semirings

D. MadhusudhanaRao, G. SrinivasaRao

Abstract— In this paper we introduced prime radicals and completely prime radicals in ternary smearing. It is proved that : If A, B and C are any three ideals of a ternary semiring T, then i) A $\subseteq$ B $\Rightarrow \sqrt{A} \subseteq \sqrt{B}$, ii) if A ∩ B∩ C ≠ Ø then $\sqrt{ABC} = \sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$ iii) $\sqrt{\sqrt{A}} = \sqrt{A}$. Further it is proved that an ideal Q of ternary semiring T is a semiprime ideal of T if and only if $\sqrt{Q}$ =Q. It is proved that if P is a prime ideal of a ternary semiring T, then $\sqrt{(P)^n}$ = P for all odd natural numbers n∈ N and if A is an ideal of a ternary semiring T then $\sqrt{A}$ ={x ∈ T: every m-system of T containing x meets A} i.e., $\sqrt{A} = \{ x \in T : M(x) \cap A \neq \emptyset \}$. Mathematical Subject Classification: 16Y30, 16Y99.

***Keywords**: left simple, lateral simple, right simple, simple, duo ternary semiring,semisimple ternary semiring, globally idempotent.*

I. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like. The theory of ternary algebraic systems was introduced by D. H. Lehmer [9]. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary groups. D. MadhusudhanaRao[9] characterized the primary ideals in ternary semigroups. about T. K. Dutta and S. Kar [6] introduced and studied some properties of ternary semirings which is a generalization of ternary rings. D. MadhusudhanaRao and G. Srinivasa Rao [11] investigated and studied about special elements in a ternary semirings. D. Madhsusudhana Rao and G. Srinivasa Rao [12, 13] introduced the ternary semiring in which satisfies the some identities and they made a study and investigated structure of certain ideals in ternary semirings. Our main purpose in this paper is to introduce the Structure of prime radical in ternary semirings.

Manuscript Received on February 2015.

Dr. D. MadhusudhanaRao, Lecturer and Head, Department of Mathematics, VSR & NVR College, Tenali, A. P. India.

G. SrinivasaRao, Asst. Prof., Department of Mathematics, Tirumala Engineering College, Narasarao Pet, A. P. India.

II. PRELIMINARIES

Definition 2.1[10] **:** A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a ***ternary semiring*** if T is an additive commutative semigroup satisfying the following conditions :

i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,
ii) $[(a + b)cd] = [acd] + [bcd]$,
iii) $[a(b + c)d] = [abd] + [acd]$,
iv) $[ab(c + d)] = [abc] + [abd]$ for all $a; b; c; d; e \in$T.

Throughout Twill denote a ternary semiring unless otherwise stated.

Note 2.2 : For the convenience we write $x_1x_2x_3$ instead of $[x_1x_2x_3]$

Note 2.3 : Let T be a ternary semiring. If A,B and C are three subsets of T , we shall denote the set ABC = $\{\Sigma abc : a \in A, b \in B, c \in C\}$.

Note 2.4 : Let T be a ternary semiring. If A,B are two subsets of T , we shall denote the set A + B = $\{a + b : a \in A, b \in B\}$.

Note2.5 : Any semiring can be reduced to a ternary semiring.

Example 2.6 : Let T be an semigroup of all $m \times n$ matrices over the set of all non negative rational numbers. Then T is a ternary semiring with matrix multiplication as the ternary operation.

Example 2.7 : Let $S = \{..., -2i, -i, 0, i, 2i, ...\}$ be a ternary semiring withrespect to addition and complex triple multiplication.

Example 2.8 : The set T consisting of a single element 0 with binary operation defined by 0 + 0 = 0 and ternary operation defined by 0.0.0 = 0 is a ternary semiring. This ternary semiring is called the ***null ternary semiring*** or the ***zero ternary semiring.***

Example2.9 : The set Q of all rational numbers with respect to ordinary addition and ternary multiplication [ ] defined by $[abc] = abc$ for all $a, b, c\in$ Q is a ternary semiring.

Definition 2.10[10]: A ternary semiring T is said to be ***commutative ternary semiring*** provided $abc = bca = cab = bac = cba = acb$for all $a,b,c\in$ T.

Example 2.11 : $(Z^0, +, .)$ is a ternary semiring of infinite order which is commutative.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

Example2.12 : The set 2I of all even integers is a commutative ternary semiring with respect to ordinary addition and ternary multiplication [ ] defined by [*abc*] = *abc* for all *a*, *b*, *c*∈ T.

Definition 2.13[14] : A nonempty subset A of a ternary semiring T is said to be ***left ternary ideal***or ***left ideal***of T if

(1) *a*, *b* ∈A implies *a* + *b*∈ A.
(2) *b*, *c*∈ T, *a*∈ A implies *bca*∈ A.

Note 2.14 : A nonempty subset A of a ternary semigroup T is a left ideal of T if and only if A is additive subsemigroup of T and TTA ⊆ A.

Example 2.15 : In the ternary semiring Z^0 , nZ^0 is a left ideal for any *n* ∈N.

Definition 2.16[14] : A nonempty subset of a ternary semiring T is said to be a ***lateral ternary ideal***or simply ***lateral ideal*** of T if

(1) *a*, *b* ∈ A implies *a* + *b*∈ A.
(2) *b*, *c*∈ T , *a*∈ A implies *bac*∈ A.

Note 2.17: A nonempty subset of A of a ternary semigroup T is a lateral ideal of T if and only if A is additive subsemigroup of T and TAT ⊆ A.

Example 2.18 : In the ternary semiring Z^0 , nZ^0 is a lateral ideal for any *n* ∈N.

Definition 2.19[14]: A nonempty subset A of a ternary semigroup T is a ***right ternary ideal*** or simply ***right ideal*** of T if

(1) *a*, *b* ∈ A implies *a* + *b*∈ A.
(2) *b*, *c* ∈ T , *a*∈ A implies *abc*∈ A.

Note 2.20: A nonempty subset A of a ternary semigroup T is a right ideal of T if and only if A is additive subsemigroup of T and ATT ⊆ A.

Example 2.21: In the ternary semiring Z^0 , nZ^0 is a right ideal for any *n* ∈N.

Definition 2.22[14] : A nonempty subset A of a ternary semiring T is a ***two sided ternary ideal*** or simply ***two sided ideal*** of T if

(1)*a*, *b* ∈ A implies *a* + *b*∈ A
(2)*b*, *c*∈ T , *a*∈ A implies *bca*∈ A, *abc*∈ A.

Note 2.23: A nonempty subset A of a ternary semiring T is a two sided ideal of T if and only if it is both a left ideal and a right ideal of T .

Example 2.24 : In the ternary semiring Z^0 , nZ^0 is a two sided ideal for any *n* ∈N.

Definition 2.25 [14]: A nonempty subset A of a ternary semiring T is said to be ***ternary ideal*** or simply an ***ideal*** of T if

(1) *a*, *b* ∈ A implies *a* + *b*∈ A
(2) *b*, *c* ∈ T , *a* ∈ A implies *bca*∈ A, *bac*∈ A, *abc*∈ A.

Note 2.26 : A nonempty subset A of a ternary semigroup T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T.

Example 2.27 : Let N be the set of all natural numbers. Define the ternary operation from $N \times N \times N \longrightarrow$ N as (*a*, *b*, *c*) = *a*.*b*.*c* where '.' is usual multiplication. Then N is a ternary semiring and A = 3N is an ideal of the ternary semiring N.

Theorem 2.28[14] : The nonempty intersection of any two ideals of a ternary semiring T is an ideal of T.

Theorem 2.29[14] : The nonempty intersection of any family of ideals of a ternary semiring T is an ideal of T.

Definition 2.30[14] :An ideal A of a ternary semiring T is said to be a ***completely prime ideal*** of T provided *x*, *y*, *z* ∈ T and *xyz*∈ A implies either *x*∈ A or *y*∈ A or *z*∈ A.

Definition 2.31[14] : Let T be a ternary semiring. A nonempty subset A of T is said to be a

c-system of T if for each *a*, *b*, *c*∈ A implies *abc*∈ A.

Theorem 2.32[14] : An ideal A of a ternary semiring T is completely prime if and only if T\A is either *c-system* of T or empty.

DEFINITION 2.33[14]: An ideal A of a ternary semiring T is said to be a ***prime ideal*** of T provided X,Y,Z are ideals of T and XYZ ⊆ A ⟹ X ⊆A or Y ⊆A or Z ⊆A.

DEFINITION 2.34[14]: A nonempty subset A of a ternary semiring T is said to be an ***m*-system** provided for any *a*, *b*, *c*∈ A implies that $T^eT^eaT^eT^ebT^eT^ecT^eT^e \cap A \neq \emptyset$.

Theorem 2.35[14] : Every completely prime ideal of a ternary semiring T is a prime ideal of T.

Theorem 2.36[14] : Let T be a commutative ternary semiring. An ideal P of T is a prime ideal if and only if P is a completely prime ideal.

Theorem 2.37[14] : Every completely prime ideal of a ternary semiring T is a completely semiprime ideal of T.

Definition 2.38[14] : An ideal A of a ternary semiring T is said to be a ***completely semiprime ideal*** provided *x*∈ T, x^n ∈ A for some odd natural number *n*>1 implies *x*∈ A.

Definition 2.39[14]: Let T be a ternary semiring. A non-empty subset A of T is said to be a ***d*-system** of T if *a*∈ A⟹ a^n ∈ A for all odd natural number *n*.

Theorem 2.40[14] : Every completely semiprime ideal of a ternary semiring T is a semiprime ideal of T.

Theorem 2.41[14]: Let T be a commutative ternary semiring. An ideal A of T is completely semiprime if and only if it is semiprime.

Definition 2.42[14]: An ideal A of a ternary semiring T is said to be ***semiprime ideal*** provided X is an ideal of T and X^n ⊆ A for some odd natural number *n*implies X ⊆ A.

Definition 2.43[14]: A non-empty subset A of a ternary semiring T is said to be an ***n*-system** provided for any *a*∈ A implies that $T^eT^eaT^eT^eaT^eT^eaT^eT^e \cap A \neq \emptyset$.

Theorem 2.44[14]: Every *m*-system in a ternary semiring

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

T is an *n*-system.

Theorem 2.45[14] : An ideal Q of a ternary semigroup T is a semiprime ideal if and only if T\Q is an *n*-system of T or empty.

Theorem 2.46[14]: If T is a globally idempotent ternary semiring then every maximal ideal of T is a prime ideal of T.

III. PRIME RADICAL AND COMPLETELY PRIME RADICAL

We use the following notation.

Notation 3.1 : If A is an ideal of a ternary semiring T, then we associate the following four types of sets.

A_1 = The intersection of all completely prime ideals of T containing A.

A_2 = $\{x \in T: x^n \in A$ for some odd natural numbers $n\}$

A_3 = The intersection of all prime ideals of T containing A.

A_4 = $\{x \in T: <x>^n \subseteq$ A for some odd natural number $n\}$

Theorem 3.2 : If A is an ideal of a ternary semiring T, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$**.**

Proof : i) $A \subseteq A_4$: Let $x \in$ A . Then $<x> \subseteq$ A and hence $x \in A_4$. Therefore $A \subseteq A_4$

ii) $A_4 \subseteq A_3$: Let $x \in A_4$. Then $<x>^n \subseteq$ A for some odd natural number *n*.
Let P be any prime ideal of T containing A.
Then $<x>^n \subseteq$ A for some odd natural number $n \Rightarrow <x>^n \subseteq$ P. Since P is prime ,$<x> \subseteq$ P and hence $x \in$ P. Since this is true for all prime ideals of P containing A, $x \in A_3$. Therefore $A_4 \subseteq A_3$

iii) $A_3 \subseteq A_2$: Let $x \in A_3$. Suppose if possible $x \notin A_2$. Then $x^n \notin$A for all odd natural number *n*.
Consider Q = $\bigcup x^n$ for all odd natural number *n*, and $x \in$ T. Let $a, b, c \in$ Q . Then $a = (x)^r$, $b = (x)^s$, $c = (x)^t$ for some odd natural numbers *r, s, t*. Therefore $abc = (x)^r (x)^s (x)^t$ = $x^{r+s+t} \in Q$ and hence Q is a *c*-system of T. By theorem 2.32, P = T\Q is a completely prime ideal of T and $x \notin P$. By theorem 2.35, P is a prime ideal of T and $x \notin P$. Therefore $x \notin A_3$.

It is a contradiction. Therefore $x \in A_2$ and hence $A_3 \subseteq A_2$.

iv) $A_2 \subseteq A_1$: Let $x \in A_2$. Now $x \in A_2 \Rightarrow x^n \in$ A for some odd natural number *n*.
Let P be any completely prime ideal of T containing A.
Then $x^n \in A \subseteq P \Rightarrow x^n \in P \Rightarrow x \in$ P. Therefore $x \in A_1$ .Therefore $A_2 \subseteq A_1$. Hence $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

Theorem 3.3 : If A is an ideal of a commutative ternary semiring T, then $A_1 = A_2 = A_3 = A_4$

Proof : By theorem 3.2, $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$. By theorem 2.36, in a commutative ternary semiring T, an ideal A is a prime ideal if A is completely prime ideal.
So $A_1 = A_3$. By theorem 2.40, in a commutative ternary semiring T an ideal A is semiprime if and only if A is completely semiprime ideal. So $A_4 = A_2$ and hence $A_1 = A_2 = A_3 = A_4$.

Note 3.4: In an arbitrary ternary semiring $A_1 \neq A_2 \neq A_3 \neq A_4$.

Example 3.5 : Let T be the free ternary semigroup generated by *a*, *b*, *c*. It is clear that A = Ta^3T is an ideal of T. Since $a^5 \in$ Ta^3T, we have $a \in A_2$. Evidently $(abc)^n \notin$ Ta^3T for all odd natural numbers *n* and thus $abc \notin A_2$. Thus A_2 is not an ideal of T. Therefore $A_1 \neq A_2$ and $A_2 \neq A_3$.

We now introduce prime radical and complete prime radical of an ideal in a ternary semiring.

Definition 3.6 : If A is an ideal of a ternary semiring T , then the intersection of all prime ideals of T containing A is called ***prime radical*** or simply ***radical*** of A and it is denoted by $\sqrt{A}$ or *rad* A.

Definition 3.7: If A is an ideal of a ternary semiring T , then the intersection of all completely prime ideals of T containing A is called ***completely prime radical*** or simply ***complete radical*** of A and it is denoted by *c.rad* A.

Note 3.8: If A is an ideal of a ternary semiring T , then *rad* A = A_3, *c.rad* A = A_1 and *rad* A $\subseteq$*c.rad* A**.**

Corollary 3.9: If $a \in \sqrt{A}$, then there exist a positive integer *n* such that $a^n \in$ A for some odd natural number $n \in$ N.

Proof : By theorem 3.2, $A_3 \subseteq A_2$ and hence $a \in \sqrt{A} = A_3 \subseteq A_2$. Therefore $a^n \in$ **A** for some odd natural number $n \in$ N.

Corollary 3.10 : If A is an ideal of a commutative ternary semiring T, then *rad* A = *c.rad* A.

proof : By theorem 3.3,*rad* A = *c.rad* A.

Corollary 3.11 : If A is an ideal of a ternary semiring T then *c.rad* A is a completely semiprime ideal of T.

proof : By theorem 2.39, *c.rad* A is a completely semiprime ideal of T.

Theorem 3.3.12 : If A, B and C are any three ideals of a ternary semiring T , then

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

i) $A \subseteq B \Rightarrow \sqrt{A} \subseteq \sqrt{B}$

ii) if $A \cap B \cap C \neq \emptyset$ then

$$\sqrt{ABC} = \sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$$

iii) $\sqrt{\sqrt{A}} = \sqrt{A}$.

proof : i) Suppose that $A \subseteq B$. If P is a prime ideal containing B then P is a prime ideal containing A. Therefore $\sqrt{A} \subseteq \sqrt{B}$.

ii) Let P be a prime ideal containing ABC. Then ABC $\subseteq$ P $\Rightarrow$ A$\subseteq$ P or B $\subseteq$ P or C $\subseteq$ P

$\Rightarrow A \cap B \cap C \subseteq$P. Therefore P is a prime ideal containing $A \cap B \cap C$.

Therefore *rad*($A \cap B \cap C$) $\subseteq$ *rad*(ABC).Now let P be a prime ideal containing $A \cap B \cap C$.

Then $A \cap B \cap C \subseteq$ P $\Rightarrow$ ABC $\subseteq A \cap B \cap C \subseteq$ P

$\Rightarrow$ ABC $\subseteq$ P. Hence P is a prime ideal containing ABC. Therefore *rad* (ABC) $\subseteq$*rad*($A \cap B \cap C$).

Therefore *rad*(ABC) = *rad*($A \cap B \cap C$).

Since $A \cap B \cap C \neq \emptyset$, it is clear that $A \cap B \cap C$ is an ideal in T. Let $x \in \sqrt{A \cap B \cap C}$.

Then there exists an odd natural number $n \in$ N such that $x^n \in A \cap B \cap C$.

Therefore $x^n \in$ A, $x^n \in$ B and $x^n \in$ C. It follows that $x \in \sqrt{A}$, $x \in \sqrt{B}$ and $x \in \sqrt{C}$. Therefore $x \in \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$.

Consequently, $x \in \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$ implies that there exists odd natural numbers $n, m, p \in$ N such that $x^n \in$ A, $x^m \in$ B and $x^p \in$ C.

Clearly, $x^{nmp} \in A \cap B \cap C$. Thus $x \in \sqrt{A \cap B \cap C}$.

Therefore if $A \cap B \cap C \neq \emptyset$then $\sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$.

iii) $\sqrt{A}$ = The intersection of all prime ideals of T containing A.

Now $\sqrt{\sqrt{A}}$ = The intersection of all prime ideals of T containing $\sqrt{A}$.

= The intersection of all prime ideals of T containing A = $\sqrt{A}$

Therefore $\sqrt{\sqrt{A}} = \sqrt{A}$.

Theorem 3.13 : If A is an ideal of a ternary semiring T then $\sqrt{A}$ is a semiprime ideal of T.

proof : By theorem 2.41 , $\sqrt{A}$ is a semiprime ideal of T.

Theorem 3.14 : An ideal Q of ternary semiring T is a semiprime ideal of T if and only if $\sqrt{Q}$ =Q.

Proof : Suppose that Q is a semiprime ideal. Clearly Q $\subseteq$ √Q.Suppose if possible √Q $\not\subseteq$ Q.

Let $a \in$ √Q and $a \notin$ Q. Now $a \notin$ Q $\Rightarrow a \in$ T\Q and Q is semiprime. By theorem 2.44,

T\Q is an n-system. By theorem 2.45, there exists an m-system M such that $a \in$ M $\subseteq$ T\Q.Q $\subseteq$ T\M and now T\M is a prime ideal of T, $a \notin$ T\M. It is a contradiction.

Therefore √Q $\subseteq$ Q. Hence √Q = Q.

Conversely suppose that Q is an ideal of S such that √Q = Q. By corollary 3.13, √Q is a semiprime ideal of T. Therefore Q is semiprime.

Corollary 3.15 : An ideal Q of a ternary semiring T is a semiprime ideal if and only if Q is the intersection of all prime ideal of T contains Q.

Proof : By theorem 3.14., Q is semiprime iff Q is the intersection of all prime ideals of T contains Q.

Corollary 3.16 : **If A is an ideal of a ternary semiring T, then √A is the smallest semiprime ideal of T containing A.**

Proof : We have that √A is the intersection of all prime ideals containing A in T. Since intersection of prime ideals is semiprime, we have √A is semiprime. Further, let Q be any semiprime ideal containing A, i.e. A $\subseteq$ Q. So √A $\subseteq$ √Q. Since Q is semiprime, By theorem 3.14, √Q = Q. Therefore √A $\subseteq$ Q. Hence √A is the smallest semiprime ideal of T containing A.

Theorem 3.17 : If P is a prime ideal of a ternary semiring T, then $\sqrt{(P)^n}$ = *P* for all odd natural numbers $n \in$ N.

Proof **:**We use induction on n to prove $\sqrt{P^n} = P$.

First we prove that $\sqrt{P} = P$. Since P is a prime ideal, $P \subseteq \sqrt{P} \subseteq P \Rightarrow \sqrt{P} = P$. Assume that $\sqrt{P^k} = P$ for odd natural number k such that $1 \leq k < n$.

Now
$$\sqrt{P^{k+2}} = \sqrt{P^k.P.P} = \sqrt{P^k} \cap \sqrt{P} \cap \sqrt{P} = \sqrt{P} \cap \sqrt{P} \cap \sqrt{P} = \sqrt{P} = P.$$

Therefore $\sqrt{P^{k+2}} = P$. By induction $\sqrt{P^n}$ = P for all odd natural number $n \in$ N.

Theorem 3.18: In a ternary semiring T with identity there is a unique maximal ideal M such that $\sqrt{(M)^n}$ = M for all odd natural numbers $n \in$ N.

Proof: Since T contains identity, T is a globally idempotent ternary semigroup.

Since M is a maximal ideal of T, by theorem 2.46 M is prime.

By theorem 3.16, $\sqrt{(M)^n}$ = M for all odd natural numbers n.

Theorem 3.19: If A is an ideal of a ternary semiring T then $\sqrt{A}$ ={$x \in$ T: every *m*-system of T containing *x* meets A} i.e., $\sqrt{A} = \{x \in T : M(x) \cap A \neq \emptyset\}$.

Proof: Suppose that $x \in \sqrt{A}$. Let M be an m-system containing x. Then T\M is a prime ideal of T and $x \notin$ T\M. If M $\cap$ A = $\emptyset$ then A $\subseteq$ T\M. Since T\M is a prime ideal containing A, $\sqrt{A} \subseteq$ T\M and hence $x \in$ T\M.

It is a contradiction. Therefore $M(x) \cap A \neq \emptyset$. Hence $x \in \{x \in T : M(x) \cap A \neq \emptyset\}$. Conversely suppose that $x \in \{x \in T : M(x) \cap A \neq \emptyset\}$. Suppose if possible

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.

$x \notin \sqrt{A}$. Then there exists a prime ideal P containing A such that $x \notin$ P. Now T\P is an *m*-system and $x \in$ T\P. A $\subseteq$ P $\Rightarrow$ T\P $\bigcap$ A = $\varnothing \Rightarrow x \notin \{x \in T : M(x) \bigcap A \neq \varnothing\}$. It is a contradiction. Therefore $x \in \sqrt{A}$.

Thus $\sqrt{A} = \{x \in T : M(x) \bigcap A \neq \varnothing\}$.

IV. CONCLUSION

In this paper mainly we studied about prime radicals in ternary semirings.

V. ACKNOWLEDGMENT

The authors would like to thank the referee(s) for careful reading of the manuscript.

REFERENCES

[1] Chinaram, R., *A note on quasi-ideal in¡¡semirings*, Int. Math.Forum, 3 (2008), 1253{1259.
[2] Dixit, V.N. and Dewan, S., *A note on quasi and bi-ideals in ternary semigroups*, Int. J. Math. Math. Sci. 18, no. 3(1995), 501{508.
[3] Dutta, T.K. and Kar, S., *On regular ternary semirings*, Advances in Algebra Proceedings of the ICM Satellite Conference in Algebra and Related Topics, World Scienti¯c, New Jersey, 2003, 3IV3{355.
[4] Dheena. P, Manvisan. S., P-prime and small P-prime ideals in semirings, International Journal of Algebra and Statistics, Volume 1; 2(2012), 89-93.
[5] Dutta, T.K. and Kar, S., *A note on regular ternary semirings*, Kyung-pook Math. J., IV6 (2006), 357{365.
[6] Kar, S., *On quasi-ideals and bi- ideals in ternary semirings*, Int. J. Math.Math.Sc., 18 (2005), 3015{3023.
[7] Lehmer. D. H., *A ternary analogue of abelian groups*, Amer. J. Math., 59(1932), 329-338.
[8] Lister, W.G., *Ternary rings*, Trans Amer. Math.Soc., 15IV (1971), 37- 55.
[9] MadhusudhanaRao. D**.,** *Primary Ideals in Quasi-Commutative Ternary Semigroups* International Research Journal of Pure Algebra – 3(7), 2013, 25IV-258.
[10] Madhusduhana Rao. D. and Srinivasa Rao. G., *A Study on Ternary Semirings*, International Journal of Mathematical Archive-5(12), Dec.-2014, 24-30.
[11] MadhusudhanaRao. D. and SrinivasaRao. G., *Special Elements of a Ternary Semirings*, International Journal of Engineering Research and Applications, Vol. IV, Issue 1(Version-5), November 201IV, pp. 123-130.
[12] MadhusudhanaRao. D. and SrinivasaRao. G., *Concepts on Ternary Semirings*, International Journal of Modern Science and Engineering Technology, Volume 1, Issue 7, 201IV, pp. 105-110.
[13] MadhusudhanaRao. D. and SrinivasaRao. G., *Characteristics of Ternary Semirings*, International Journal of Engineering Research and Management – Vol.2, Issue 01, January 2015, pp. 3-6.
[14] MadhusudhanaRao. D. and SrinivasaRao. G.,*Structure of Certain Ideals in Ternary Semirings*-International Journal of Innovative Science and Modern Engineering(IJISME)-Volume-3, Issue-2, January, 2015.

Dr D. MadhusudhanaRao, completed his M.Sc. from Osmania University, Hyderabad, Telangana, India. M. Phil. from M. K. University, Madurai, Tamil Nadu, India. Ph. D. from AcharyaNagarjuna University, Andhra Pradesh, India. He joined as Lecturer in Mathematics, in the department of Mathematics, VSR & NVR College, Tenali, A. P. India in the year 1997, after that he promoted as Head, Department of Mathematics, VSR & NVR College, Tenali. He helped more than 5 Ph.D's and at present he guided 5 Ph. D. Scalars and 3 M. Phil., Scalars in the department of Mathematics, AcharyaNagarjuna University, Nagarjuna Nagar, Guntur, A. P.A major part of his research work has been devoted to the use of semigroups, Gamma semigroups, duo gamma semigroups, partially ordered gamma semigroups and ternary semigroups, Gamma semirings and ternary semirings, Near rings ect. He acting as peer review member to the "***British Journal of Mathematics & Computer Science".*** He published more than **38 research papers** in different International Journals in the last three academic years.

G. Srinivasarao, He is working as an Assistant Professor in the Department of Applied Sciences & Humanities, Tirumala Engineering College. He completed his M.Phil. inMadhuraiKamaraj University . He is pursuing Ph.D. under the guidance of Dr.D.Madhusudanarao in AcharyaNagarjuna University. He published more than 6 research papers in popular international Journals to his credit. His area of interests are ternary semirings, ordered ternary semirings, semirings and topology. Presently he is working on Ternary semirings

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication Pvt. Ltd.